THE DEMAND FOR TRAVEL AND FOR TRAVELCARDS ON LONDON REGIONAL TRANSPORT

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1. INTRODUCTION

Londoners have the choice of paying for their bus and underground travel either by purchase of an "ordinary" ticket, valid for one trip, or by purchase of a farecard, which is valid for a period of time with no extra charge for trips taken.[1] The main farecard currently issued is the Travelcard, which also has the attraction of allowing free interchange between the bus and underground (and since January 1989 the British Rail) networks. In this paper we develop a simple model in which decisions to travel are jointly determined with decisions to purchase a particular form of ticket (ordinary or farecard). Our modelling emphasis is on clearly setting out the assumptions underlying the model, and on testing them against the data. The result, we believe, is a model which is consistent with demand theory and, at the same time, gives an acceptable characterisation of the historical data. To that extent it will be useful in forecasting and policy analysis. We illustrate this latter function by using the model to derive an indicative simulation of the possible effects of the decision by London Regional Transport (LRT) to introduce Travelcards in May 1983.

Our model draws heavily on a sequence of earlier models developed by the Group Planning Office of London Regional Transport, the most recent of which is reported in a document (LRT, 1987) generally referred to as R266. In particular,

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[1] We classify one-day travelcards as ordinary tickets.

we follow R266 in using a sample of four-weekly data (that is, thirteen observations per year).[2]

Our approach is to see travellers as making a two-stage decision: at the first stage they decide whether to travel, and at the second they decide how to pay for that travel. Imposition of separability on these decisions implies that the overall travel quantum will be described by a standard demand function dependent on the price of travel and the level of Central London employment (the major demand shift variable), while the decision to purchase a farecard will depend on the relative prices of ordinary tickets and farecards. Farecard purchase will only depend on employment and the price of travel through the overall travel quanta. This conjecture seems broadly confirmed by the data.

We model purchases of bus and underground ordinary ticket jointly with purchases of farecards. This implies a model with six equations (bus ordinary travel, underground ordinary travel, underground season ticket validity, bus pass validity, Travelcard validity, and Capitalcard validity). In fact, we also estimate a seventh equation to take into account the fact that bus miles run, a measure of the availability of bus services, is in principle jointly determined with the bus demand variables.

The bus and underground quantum equations, which form the backbone of the model, are estimated over a sample of 205 four-weekly observations covering the sample period January 1972 to October 1987. (The farecard equations are estimated over shorter samples reflecting the different ticketing arrangements in force during different subperiods.) Equation specifications are derived from a theoretical model, outlined in section 2, applied flexibly to the data to give an estimated model which is both consistent with the theory but also satisfied by the data. The theory implies a relatively strong structure for the two ordinary ticket (quantum) equations, but considerably less structure for the farecard equations. But for both sets of equations there is a large element of discretion, both in choice of explanatory variables and in dynamic specification. Throughout we followed the "general to simple" methodology in which the model equations are initially specified in very general terms, and are simplified so that there is no significant loss of fit from the unrestricted specifications.

In fact, the model we develop in section 2 implies five separate sets of restrictions. These are:

(a) Intra-equation restrictions, arising from the travel quantum model, on the distributed lags of the farecard variables in the ordinary ticket equations.

(b) Intra-equation restrictions, arising from the simplification process, on the price coefficients in the ordinary ticket equations.

(c) Inter-equation restrictions, arising from demand theory, on the cross-price coefficients in the ordinary ticket equations.

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[2] Our modelling approach differs from that in R266 in three substantive and in three methodological respects. Substantially, we differ from R266 in (i) the treatment of farecards, (ii) the treatment of the response to price over time, and (iii) the choice of demand shift variables. Methodologically, we differ from R266 in (iv) the treatment of seasonality, (v) our adoption of a "general to simple" testing procedure in the choice of equation specification, and (vi) our choice of estimation methods.

(d) Variable exclusion restrictions, arising from the travel quantum model, relating to the variables included in the farecard equations.

(e) Standard specification tests with regard to serial correlation on each of the equations in the model (see for example Harvey, 1990, chapter 5).

With 205 observations, these tests amount to a very strong test on the validity of the model.

2. MODEL STRUCTURE

Ordinary ticket purchases

LRT has not collected trip data on a continuing basis. Trip data have therefore to be inferred from revenue receipts, but this is complicated by the existence of farecards. LRT allocates farecard revenues between the bus and underground networks on the basis of survey data.[3] The data analysed in R266 are the revenues from all sources which are attributed to the bus and underground networks and deflated by appropriated price indices of bus and underground services respectively. These data therefore amalgamate the effects of travel covered by ordinary tickets and of that covered by farecards. We have attempted to distinguish these separate modes of payment by constructing travel quanta both for ordinary travel and for farecard validity. This has meant that we have had to return to the original revenue records to obtain separate series for sales of ordinaries and of farecards.

Let y_{jt} be the quantum of ordinary tickets ("ordinaries") purchased in period t for travel mode j (j = *b*us, *u*nderground). These quanta are calculated by deflating revenue R_{jt} by the mode j ordinary price index p_{jt}.

$$y_{jt} = R_{jt}/p_{jt} \qquad (j = b,u) \tag{1}$$

In this study the price indices are based on the 1983 Central Zone fare of 40p; this enables us to interpret the travel quanta as an equivalent number of Central Zone trips.[4]

Farecards

We distinguish four types of farecards. These are (1) underground season tickets, (2) bus passes, (3) Travelcards, and (4) Capitalcards. We ignore concessionary tickets (such as free cards for the elderly) and transfers between LRT and British Rail ("clearances"). Travelcards, which allow free travel on both bus and underground networks in the specified zones, were introduced in mid-1983, at which point underground season tickets were no longer issued. Capitalcards were introduced in January 1985 and allow free transfer between the British Rail and LRT

[3] Also British Rail in the case of Capitalcards and, from January 1989, also Travelcards. In addition, LRT receives revenue, referred to as "clearances", from British Rail with respect to certain sales made at British Rail stations.

[4] London has been divided into a number of uniform fare zones for purposes of ticketing, since 1980 on a preliminary basis and fully since 1983. For data definitions and methods of construction see Gilbert and Jalilian (1989), appendix.

networks. In January 1989, Travelcards and Capitalcards were amalgamated under the name Travelcard but were given validity on British Rail. In effect, Travelcards became Capitalcards.

Let n_{it} be the number of farecards of type i ($i = 1, \ldots, 4$) valid in period t (that is, farecard i "validity" in period t). Write y^*_{jt} for the latent travel quantum on mode j ($j = b,u$) in period t, interpreted as the amount of travel that would have been made in the absence of all farecards. We suppose

$$y_{jt} = y^*_{jt} - \sum_{i=1}^{4} \theta_{ji} n_{it} \qquad (j = b,u) \tag{2}$$

An approximate measure of the number of trips made on the network by passengers with farecards may be obtained by attaching a conversion factor to the numbers of each farecard valid in each period ("validities"). Thus, if a typical Travelcard purchaser makes ten underground trips and two bus trips per week, this will imply conversion factors of 40 and 8 respectively for underground and bus on the four-weekly basis of our data series. The R266 data may be seen as being implicit travel quanta, using attributions derived from survey data. Our procedure is to estimate these conversion factors by measuring the loss of ordinary ticket sales resulting from a marginal increase in the numbers of each type of farecard sold. We then use these estimated conversion factors to construct a measure of the underlying quanta of bus and underground travel.

There are two difficulties in interpreting the resulting travel quanta as measures of the numbers of trips. The first results from the fact that our estimated conversion factors relate to the marginal farecard purchaser, whereas the conversion factors appropriate to constructing an index of the number of trips relate to the representative or average farecard purchaser. It must be expected that the marginal purchaser will make substantially less use of his/her card than will the average purchaser, particularly if convenience is an important motive for farecard purchase. Secondly, purchase of a farecard enables one to travel at zero marginal financial cost, and it therefore appears likely that an increase in the number of farecards in use will result in an increase in the number of trips made. The LRT survey data relate the number of trips made by farecard holders, but our procedure will estimate the number of ordinary trips *displaced* by purchase of a farecard, where θ_{ji} is the marginal displacement effect of farecard i on ordinary ticket purchases on travel mode j. We should in general expect that the estimated displacement effects will underestimate the number of trips undertaken, even by the marginal traveller. We take these displacement effects to be constant over time.

At different times during our sample period different types of ticketing arrangements have been in force. In looking for a structural econometric model which remains valid over these different arrangements, it is natural to formulate one's model in terms of the unobserved quanta y^*_{jt} that would have applied in a no-farecard regime. Write $y_t' = (y_{bt}, y_{ut})$, $y^{*\prime}_t = (y^*_{bt}, y^*_{ut})$, $n_t' = (n_{1t}, \ldots, n_{4t})$, and let x_t be a vector of forcing variables (prices, incomes, travel conditions, etc.). Then a general dynamic log-linear simultaneous model for y^* is given as

$$B(L)\ln y^*_t = \Gamma x_t + u_t \tag{3}$$

where u_t is a vector of disturbances and $B(L)$ is a matrix polynomial in the lag

operator L, defined so that $Lx_t = x_{t-1}$. Using the approximation $\ln(1 + z) = z$, which is accurate for small values of z, we may approximate (2) as

$$\ln y_{jt} = \ln y^*_{jt} - \sum_{i=1}^{4} \theta_{ji} \frac{n_{it}}{\bar{y}_j} \qquad (j = b,u) \tag{4}$$

where $\bar{y}_j$ is the sample mean of y_{jt}. Substituting (4) into (3) eliminates the latent quantum y^* to give

$$B(L)\ln y_t = -B(L)\Theta' n^*_t + \Gamma x_t + u_t \tag{5}$$

where Θ is the matrix

$$\Theta = \begin{bmatrix} \theta_{b1} & \theta_{u1} \\ \theta_{b2} & \theta_{u2} \\ \theta_{b3} & \theta_{u3} \\ \theta_{b4} & \theta_{u4} \end{bmatrix} \text{ and } n^*_t = \begin{bmatrix} \bar{y}_u & 0 & 0 & 0 \\ 0 & \bar{y}_b & 0 & 0 \\ 0 & 0 & \bar{y}_u & 0 \\ 0 & 0 & 0 & \bar{y}_b \end{bmatrix}^{-1} n_t$$

The framework defined by equation (5) is very general, and it is desirable to impose a somewhat greater degree of structure on the model. We do this by specifying

$$B(L) = \begin{bmatrix} 1 - \beta_b(L) & 0 \\ 0 & 1 - \beta_u(L) \end{bmatrix} \tag{6}$$

where

$$\beta_j(L) = \beta_{j1} L + \beta_{j2} L^2 + \ldots \qquad (j = b,u)$$

This specification rules out both contemporaneous and dynamic interactions between the bus and underground quanta. Equation (5) may now be expressed as

$$\ln y_{jt} = \beta^*_j(L)\ln y_{j,t-1} + \sum_{i=1}^{4} \theta_{ji} \left[n^*_{it} - \beta^*_j(L) n^*_{i,t-1} \right] + \gamma'_j x_t + u_{jt} \qquad (j = b,u) \tag{7}$$

Comparing (7) with its unrestricted counterpart

$$\ln y_{jt} = \beta^*_j(L)\ln y_{j,t-1} + \sum_{i=1}^{4} \phi_{ji}(L) n^*_{it} + \gamma'_j x_t + u_{jt} \qquad (j = b,u). \tag{8}$$

we note that our specification imposes the restrictions

$$\phi_{ji}(L) = \theta_{ji}\beta_j(L) \qquad (i = 1, \ldots, 4; j = b,u) \tag{9}$$

These restrictions (set (a) in section 1) give proportionality between the farecard lag polynomials and the lag polynomial on the dependent variable of the equation in question. Experimentation revealed that it is possible to restrict the lag polynomials $\beta_b(L)$ and $\beta_u(L)$ to be respectively first and second order. It did not prove possible to determine six farecard coefficients accurately, and so the matrix θ was simplified to

$$\Theta = \begin{bmatrix} 0 & \theta_{u1} \\ \theta_{b2} & 0 \\ \theta_{b3} & \theta_{3u} \\ \theta_{b4} & 0 \end{bmatrix} \tag{10}$$

The restriction $\theta_{b1} = 0$ implies that the marginal underground season ticket holder would not reduce his/her use of bus and, correspondingly, the restriction $\theta_{u2} = 0$ implies that the marginal bus pass holder would not reduce his/her use of underground.

The restriction $\theta_{u4} = 0$, which is clearly supported by the data, appears more problematic, since it appears to imply that Capitalcard holders did not use the underground network, an implication which was contradicted by casual observation. Note, however, that, before the introduction of Capitalcards in 1985, British Rail season tickets were valid through to specified underground stations. The introduction of Capitalcards generalised this validity (i) to the entire Central Zone underground network and (ii) to LRT buses. The net impact of the introduction of Capitalcards was therefore a reduction in receipts on bus but not on underground services, since Capitalcard holders had not in any case been purchasers of ordinary underground tickets.

A complication is that we do not have data on validity of bus passes and underground season tickets through the entire sample.

We experimented by attempting to construct the missing values from auxiliary regressions, but in the end preferred to replace these series throughout the entire sample by quanta of bus passes and underground season tickets. The quanta were calculated in the same way as in (1) from revenue attributable to valid bus passes and underground seasons, deflated by a lag distribution of the respective prices; the lags reflect the average number of annual, quarterly, monthly and weekly tickets. These quanta, which are calculated on a numbers equivalent basis, replace the underground season ticket numbers and bus pass numbers respectively in (7), and this complicates interpretation of the parameters θ_{u1} and θ_{b2}. There is no problem with the Travelcard and Capitalcard validity variables n_{3t} and n_{4t}. In addition, we set $\theta_{b4} = 0$ for the first six four-week periods of 1985, since the impact on reduced bus receipts is not apparent in the first half of 1985. This suggests that Capitalcard holders may not have taken advantage of the bus validity of their cards during this initial period.

We turn now to specification of the vector x_t of forcing variables. Write

$$\gamma_j' x_t = \rho_j' s_t + \alpha_j' p_t + \delta_j' z_t \qquad (j = b,u) \qquad (11)$$

where s_t is a vector of service variables, p_t is a vector of prices and z_t is a vector of strictly exogenous variables. We consider each of these sets of variables in turn.

Service variables

Candidates for the service variables are (i) measures of the extent of services offered or provided; (ii) measures of failure to provide services, typically as the result of strike action; and (iii) measures of service quality. We experimented with mixed success with all of these. Data are available for the bus network on both miles scheduled and miles run. At times the number of miles scheduled has been optimistic, and this has subsequently resulted in sharp downward revision of scheduled miles without any effect on miles run. This variable therefore does not appear very useful in the statistical analysis. On the other hand, we did find that the bus miles run variable (*BSMR*) is important in explaining demand for bus

travel. We were unable, however, to find any comparable effect from inclusion of a variable measuring underground miles run. We allowed for possible endogeneity of bus miles run by estimating using an Instrumental Variables (*IV*) estimator.

Strike variables are straightforward, and we included variables measuring bus days lost (*BSDL*) (that is, the number of days on which there was a significant disruption) in the bus equation and underground days lost (*UGDL*) in the underground equation. In addition, it was important to include a dummy variable for the period 1982:7 (*DUM*827) when strike action simultaneously affected the underground and British Rail networks. We had less success with service quality. LRT kindly made available to us a number of variables measuring waiting time for buses: these variables typically entered the bus quantum equation with the correct sign but the level of significance was low. They were therefore omitted.

Price variables

Turning now to the price variables, we considered current and lagged (log) prices relating to the bus, underground and British Rail networks (respectively *BSP*, *UGP* and *BRP*), in each case deflated by the Retail Price Index.[5] We found it acceptable to simplify the lag distributions in terms of the current prices, the prices lagged one period, and the annual average of the prices over the previous year. This gives a set of nine α coefficients in each equation. Inspection of the unrestricted coefficients suggested further simplification, reflected in (12b) and (12u). Thus, in the bus equation, we use

$$\begin{aligned}\alpha'_b p_t = {} & \alpha_{b1}\ln(BSP_t/UGP_t) + \alpha_{b2}\ln(BSP_t/RPI_t) + \alpha_{b3}\Delta\ln(BSP_t/RPI_t) \\ & + \alpha_{b4}\ln(BSP_t/BRP_{t-1}) + \alpha_{b5}\ln(BSP^a_{t-13}/UGP^a_{t-13}) \\ & + \alpha_{b6}\ln(BSP^a_{t-13}/BRP^a_{t-13})\end{aligned} \tag{12b}$$

while in the underground equation we have

$$\begin{aligned}\alpha'_u p_t = {} & \alpha_{u1}\ln(BSP^*_t/UGP_t) + \alpha_{u2}\ln(UGP_t/RPI_t) + \alpha_{u3}\Delta\ln(UGP_t/RPI_t) \\ & + \alpha_{u4}\ln(UGP_t/BRP_{t-1}) + \alpha_{u5}\ln(BSP^a_{t-13}/UGP^a_{t-13}) \\ & + \alpha_{u7}\ln(BRP^a_{t-13}/RPI^a_{t-13})\end{aligned} \tag{12u}$$

where

$$\ln BSP^*_t = \tfrac{1}{2}(\ln BSP_t + \ln BSP_{t-1}).$$

These specifications give a convenient separation of impact effects of price changes (picked up by the first difference of the own price), short-run effects

[5] We experimented also with the inclusion of variables measuring the petrol price, as a possible indicator of substitution of private for public transport, and of real earnings, as a measure of the price of time. The petrol price variable was always highly insignificant, whereas the earnings variable when significant was implausibly signed. These variables were therefore omitted.

(reflected in the coefficients of the current price variables), and longer-run effects (modelled through the annually lagged annual average price variables). In each case, this specification was tested against the unrestricted specification

$$\alpha'_j p_t = \sum_{k=0}^{1} a_{jk1} \ln\left(\frac{BSP_{t-k}}{RPI_{t-k}}\right) + a_{jk2} \ln\left(\frac{UGP_{t-k}}{RPI_{t-k}}\right) + a_{jk3} \ln\left(\frac{BRP_{t-k}}{RPI_{t-k}}\right) + a_{j21} \ln\left(\frac{BSP^a_{t-13}}{RPI^a_{t-13}}\right) + a_{j22} \ln\left(\frac{UGP^a_{t-13}}{RPI^a_{t-13}}\right) + a_{j23} \ln\left(\frac{BRP^a_{t-13}}{RPI^a_{t-13}}\right) \tag{13}$$

This set of restrictions corresponds to (b) in the discussion in section 1.

The major implication of consumer demand theory for our model is the Slutsky symmetry (integrability) condition.

$$\frac{\partial y_u}{\partial p_b} = \frac{\partial y_b}{\partial p_u} \tag{14}$$

where p_b is the price of bus (ordinary) tickets and p_u is the price of underground (ordinary) tickets. We test the Slutsky restriction (14) on the long-run solution of (7). This is sensible, since if travellers are adjusting towards their equilibrium choices, as indicated by the presence of the lagged dependent variables, there is no reason to suppose that short-run decisions would satisfy the integrability condition. A minor complication is that our model is specified in logarithmic terms, while the Slutsky condition relates to variables measured in levels. We follow the standard approach of imposing (14) at the sample mean of the data. In terms of the α coefficient of (12b,u) and the distributed lag coefficients of (6) this implies

$$\frac{\alpha_{b1} + \alpha_{b5}}{1 - \beta_{b1}} = -k \frac{\alpha_{u1} + \alpha_{u5}}{1 - \beta_{u1} - \beta_{u5}} \tag{15}$$

where

$$k = \frac{\overline{BSQ}/\overline{UGQ}}{\overline{BSP}/\overline{UGP}}$$

This restriction corresponds to (c) in the discussion in section 1.

The vector z_t of strictly exogenous variables consists of a constant term, a complete set of (twelve) seasonal dummies, weather variables, an interaction term, Greater London population, a measure of employment in Central London and an index of tourist visitor nights in Great Britain (*VIS*).[6] The elasticity of travel demand with respect to Greater London population (*POP*) is poorly determined in both bus and underground equations, and in any case produces collinearity with the time trend. We therefore chose to restrict the long-run elasticity of this variable to be unity in both equations.

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[6] There is no specific information on nights spent by tourist visitors in Central London.

Employment

It is widely believed that the rise in travel demand in London over the past five years is the result of a substantial increase in employment in Central London (see for example Evans and Crampton, 1989). Confirmation of this view is made difficult by the fact that, over the past decade, the only figures for Central London employment calculated by the Department of Employment have been those derived from the triennial Census of Employment, which has in any case now been put on to a sample basis, and which reports only after a delay of nearly three years. Although these data give a detailed picture of employment patterns at a date in the fairly recent past, they are of little use in statistical time series analysis using monthly data; and the long reporting delay makes them virtually useless in forecasting. Nor is it possible to obtain estimates of employment by area from the very detailed and rapidly produced figures on unemployment by area, since unemployment is measured on the basis of place of residence, whereas employment is measured on the basis of place of work.[7]

The authors of R266 attempted to circumvent this problem by using the UK retail sales variable to shift the travel demand curves over time. However, this variable contributes little to the explanation of the demand for bus travel, and our experience was that its inclusion in the underground equation resulted in a number of estimated interaction effects which we believe to be spurious. We concluded that there was no alternative to construction of a measure of employment in Central London. The closest we were able to get to this was a measure (*SEEMP*) of employment in the South East, which we constructed by deflating the annually published measure of all employment income in the South East, taken from the regional economic accounts, by the UK measure of average earnings.

Since this work was completed, the 1987 census of employment figure for employment in Central London has become available. It suggests that our series has tended to exaggerate the growth of employment in the latter half of the eighties. There are two reasons why our constructed variable may give a misleading measure of Central London employment. First, the fact that average earnings in Central London are considerably higher than those in the remainder of the United Kingdom will impart a downward bias to the estimated level of employment. Provided the relativity between earnings in Central London and in the remainder of the UK remains constant, this will affect neither the forecasting performance of our model nor the estimated employment elasticities. But if this relativity changes, or if employment in Central London moves discrepantly from that in the remainder of the South East, our estimates and forecasts will be biased. It is difficult to argue that there has been complete uniformity in both these respects over the past decade; nevertheless, we believe that this measure gives a more reliable guide to changes in employment in Central London than any other available at the time we did this work.

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[7] Changes in the tiny figure relating to unemployment in the City of London are, for example, completely uninformative about growth in employment in the City.

Seasonality

The decision to model seasonality by a set of constant shift intercepts was taken after some experimentation with alternatives. In particular, we found that this approach gave a better fit (in terms of equation standard errors) than use of the annual difference (Δ_{13}). Nevertheless, it is clear that seasonality effects are not completely constant over the sample (1972–87). There are two particular problems. Firstly, the merged Christmas and New Year holiday has led to significantly longer breaks over the New Year period. This has resulted in a reduction in work-related travel but an increase in leisure-related travel over this period; and, as a consequence, the seasonal effect in the initial period of the year is very sensitive to the weather, particularly the temperature. This is picked up in the underground equation by an interaction term between the first period dummy and the temperature, phased in from 1977. Secondly, the variability of the date of the Easter holiday results in non-constancy of the seasonal effects for the fourth and fifth four-week periods of the year.

The bus miles run equation was specified as a partial adjustment responding to the change in the bus ordinary quantum, and to changes in bus fares. The following appeared to work reasonably well:

$$\begin{aligned}\ln BSMR_t = {} & \lambda \ln BSMR_{t-1} + \mu_1 \Delta \ln BSQ_t + \mu_2 \Delta \ln(BSP/RPI)_t \\ & + \mu_3 \ln(BSP_{t-1}/AVER_{t-1}) + \mu_4 t + \mu_5 (BHOLS_t - \lambda BHOLS_{t-1}) \quad (16) \\ & + \text{constant} + \text{seasonals}.\end{aligned}$$

The average earnings ($AVER$) variable is included as a measure of LRT labour costs. The final term effectively corrects the bus miles run variable for the effect of the number of bank holidays ($BHOLS$) during the period.

3. THE ESTIMATED ORDINARY TICKETS MODEL

The bus and underground ordinary equations (7) were estimated jointly with the bus miles equation (16), using Nonlinear Three Stage Least Squares (NL3SLS), which is asymptotically equivalent to Full Information Maximum Likelihood (FIML). This method of estimation allows imposition of the within-equation restrictions (9, 12b, 12u) and the cross-equation restrictions (15), while at the same time accommodating potential simultaneity through the bus miles run variables.

The most general equations estimated were:

$$\begin{aligned}\ln y_{bt} = {} & \beta_{b1} \ln y_{b,t-1} \\ & + \theta_{b20} n^*_{2t} + \theta_{b21} n^*_{2,t-1} + \theta_{b22} n^*_{2,t-2} + \theta_{b23} n^*_{2,t-3} \\ & + \theta_{b30} n^{**}_{3t} + \theta_{b31} n^{**}_{3,t-1} + \theta_{b32} n^{**}_{3,b-2} \theta_{b33} n^{**}_{3,t-3} \\ & + \theta_{b40} n^*_{4t} + \theta_{b41} n^*_{4,t-1} + \theta_{b42} n^*_{4,t-2} + \theta_{b43} n^*_{4,t-3} \\ & + \rho'_b s_t + \alpha'_b p_t + \delta'_b z_t\end{aligned} \quad (17b)$$

and

$$\begin{aligned}\ln y_{ut} = {} & \beta_{u1}\ln y_{u,t-1} + \beta_{u2}\ln y_{u,t-2} \\ & + \theta_{u10}n^*_{1t} + \theta_{u11}n^*_{1,t-1} + \theta_{u12}n^*_{1,t-2} + \theta_{u13}n^*_{1,t-3} \\ & + \theta_{u30}n^*_{3t} + \theta_{u31}n^*_{3,t-1} + \theta_{u32}n^*_{3,t-2} + \theta_{u33}n^*_{3,t-3} \\ & + \rho'_u s_t + \alpha'_u p_t + \delta'_u z_t\end{aligned} \tag{17u}$$

Here

$$n^{**}_{3t} = {}^1/_3(n^*_{3t} + n^*_{3,t-1} + n^*_{3,t-2})$$

This substitution was motivated by the existence of significant lagged effects of bus prices on purchases of ordinary bus tickets. In terms of our model, this suggests either that bus pass validity has been inaccurately calculated, or that bus passes are improperly used beyond the period of their validity.

We employed a nested hypothesis testing structure (see for example Mizon, 1977) which allows the sequential imposition of the hypotheses of interest. This structure is displayed, with the test results, in Table 1. The initial set of restrictions imposed on (17b,u) arises from the fact that the predetermined variables in the model appear insufficiently informative to allow us to identify well-determined current period effects for the farecard variables on the travel quanta. We therefore had to compromise by introducing a theoretically unjustified one-period lag in the underground season ticket and Travelcard variables. There is no problem with bus passes or Capitalcards. The implied restrictions are:

$$\theta_{b30} = \theta_{u10} = \theta_{u30} = 0 \tag{18}$$

which give a χ^2_3 value of 7.01. This is marginally accepted; and, since the three restricted coefficients are estimated with positive coefficients in the unrestricted estimates (A), we choose to regard this marginal acceptance as reflecting lack of identification of these three current farecard coefficients rather than as a rejection of our theoretical approach.

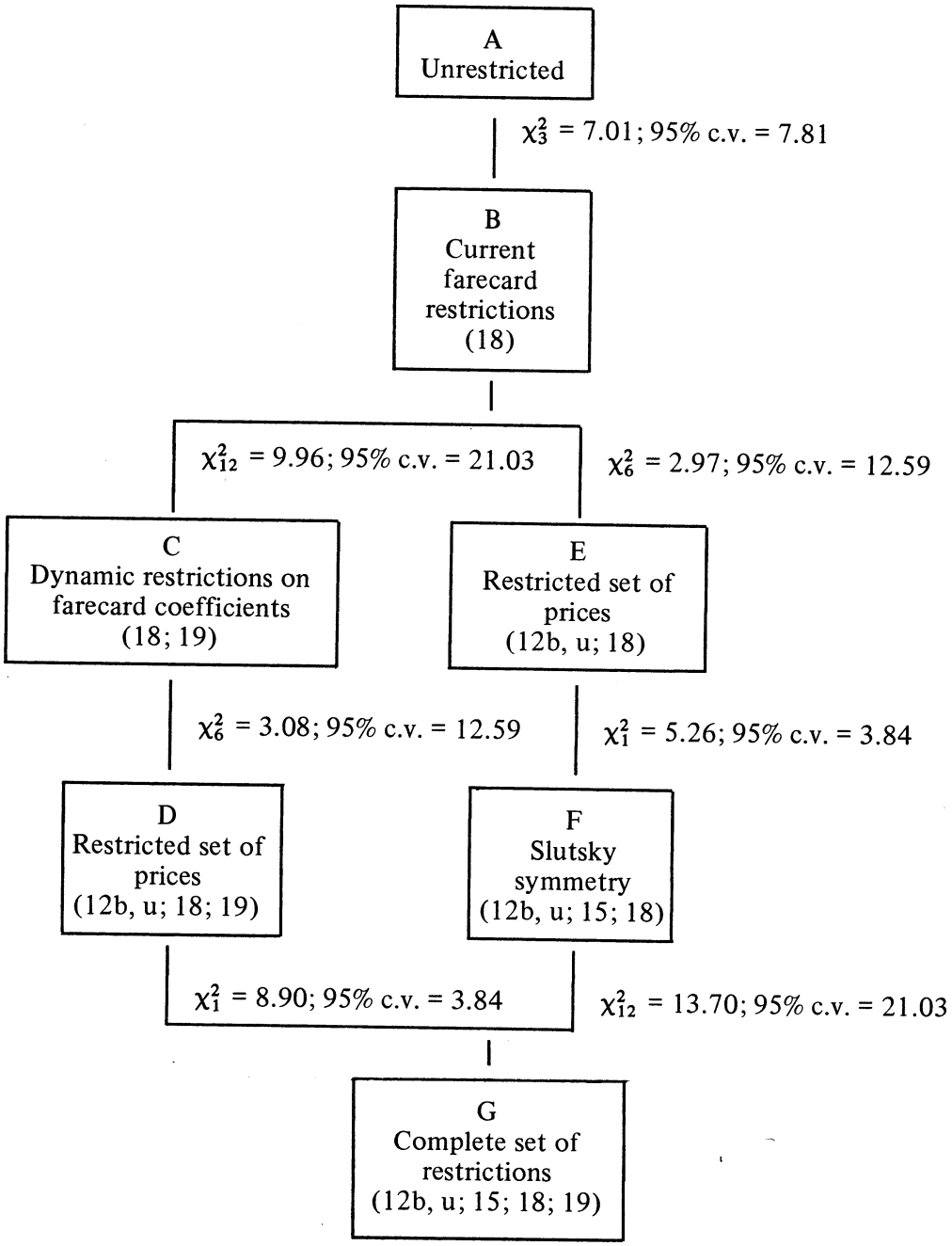
At this point the procedure branches. On the left-hand branch we impose the restriction implied by (9) on the lagged farecard coefficients. Specifically, (9) implies the twelve restrictions:

$$\begin{aligned}
\theta_{b21} &= (1-\beta_{b1})\theta_{b20} & \theta_{b42} &= 0 \\
\theta_{b22} &= (1-\beta_{b1})\theta_{b20} & \theta_{b43} &= 0 \\
\theta_{b23} &= -\beta_{b1}\theta_{b20} & \theta_{u12} &= -\beta_{u1}\theta_{u11} \\
\theta_{b32} &= -\beta_{b1}\theta_{b31} & \theta_{u13} &= -\beta_{u2}\theta_{u11} \\
\theta_{b33} &= 0 & \theta_{u32} &= -\beta_{u1}\theta_{u31} \\
\theta_{b41} &= -\beta_{b1}\theta_{40} & \theta_{u33} &= -\beta_{u2}\theta_{u31}
\end{aligned} \tag{19}$$

The associated likelihood ratio test gives a value of $\chi^2_{12} = 9.96$, which is clearly acceptable. Continuing down the left-hand arm of Table 1, we now restrict the coefficients of the price variables in accordance with (12b,u), giving a test value of $\chi^2_6 = 3.08$, and finally impose the Slutsky symmetry condition (15), giving a test value of $\chi^2_1 = 8.90$. The right-hand arm of the table reverses the order of the test, first imposing the two sets of reductions on the price coefficients and finally imposing the restrictions on the lagged farecard coefficients.

TABLE 1

Tests of the Model Restrictions



Numbers in parentheses indicate equations specifying restrictions imposed.
Sample: 1972.1 → 1987:10.

In both cases, the simplification of the set of price variables implied by (12b,u) is clearly accepted, while the Slutsky symmetry condition is rejected at any reasonable test size. The Slutsky restriction is rejected because (15) requires that the bus price elasticity in the underground quantum equation should only be slightly less than the underground elasticity in the bus equation ($k = 0.9$), whereas the unrestricted estimates imply substantially greater substitutability in the bus equation (see Tables 3 and 4 below). Rejection of the symmetry restriction focuses attention of the left-hand arm of Table 1. We therefore choose to work with specification D, which restricts the lagged farecard coefficients and simplifies the vector of prices without imposing symmetry.

The results from estimation of specification D are given in Table 2. The estimated values of the farecard usage coefficients (the θs) show that, in the absence of farecards, the marginal underground season ticket displaced approximately 11 equivalent Central Zone tube trips per week, the marginal bus pass displaced 33 equivalent Central Zone bus trips, the marginal Travelcard displaced 16 equivalent Central Zone tube trips, and the marginal Capitalcard displaced 17 equivalent Central Zone bus trips. Note also that the estimates suggests that, at the margin, the sale of an additional Travelcard generated five additional equivalent Central Zone bus trips. This suggests that Travelcard users are primarily underground travellers, but confirms suggestions from survey work that Travelcards may generate additional paid bus trips (Jones, 1987).

It is natural to wish to interpret these marginal displacement effects as usage coefficients; but this translation is dangerous. In 1982, the conversion factor for underground season ticket and bus pass numbers to quanta was in both cases one fifth (see Gilbert and Jalilian, 1989, appendix), suggesting that the average underground season and bus pass holder would make 50 equivalent Central Zone trips per week. It is to be expected that the marginal traveller will make less use of the facility. Nevertheless the estimates (with the exception of that for bus passes) appear too low to be plausible as usage coefficients, even for the marginal farecard purchaser.

The Lagrange Multiplier thirteenth (annual) order tests indicate the presence of serial correlation. We believe this to be due to non-constant seasonality. This view is supported by the fact that fourth order tests fail to indicate any trace of residual serial correlation.

The impact and equilibrium elasticities implied by the estimates reported in Table 2 are tabulated in Tables 3a and 3b, respectively. Note that, though the equations estimated in Table 2 appear to hold the farecard variables constant, and therefore appear to omit the effects of price changes on farecard validity, the theoretical specification (3) makes it clear that these elasticities may legitimately be interpreted as the elasticities of the overall travel quanta (y^*) with respect to changes in prices. The demand for bus travel is very much more price-responsive than is the demand for underground travel; and both sets of demand elasticities are higher in the long run than in the short run.[8] Indeed, in the long run, bus travel is demanded elastically (that is, its own-price elasticity exceeds unity); this

[8] But it is notable that there is little change in the elasticities with respect to the British Rail price.

TABLE 2

Estimated Ordinary Quanta Equations

	ln(*BSQ/POP*)	ln(*UGQ/POP*)
β_{j1} (dep. var.)$_{-1}$	0.195	0.337
	(0.063)	(0.053)
β_{j2}(dep. var.)$_{-2}$	–	0.138
		(0.042)
$\theta_{j1}[1-\beta_j(L)]\,UGS_{-1}/\overline{UGQ}$	–	−10.54
		(4.98)
$\theta_{2j}[1-\beta_j(L)](BSS+BSS_{-1}+BSS_{-2})/3\overline{BSQ}$	−33.41	–
	(6.32)	
$\theta_{3j}[1-\beta_j(L)]\,TCDS_{-1}/\overline{UGQ}$	5.03	−15.89
	(1.60)	(2.85)
$\theta_{4j}[1-\beta_j(L)]\,CAPS/\overline{BSQ}$	−17.37	–
	(3.54)	
ρ_{j1} ln(*BSMR/POP*)	0.581	–
	(0.104)	
ρ_{j2} *BSDL*/28, *UGDL*/28	−0.447	−0.028
	0.184	(0.054)
ρ_{j3} *DUM*827	0.114	−0.335
	(0.035)	(0.030)
α_{j1} ln(*BSP/UGP*), [ln(*BSP/UGP*) + ln(*BSP*$_{-1}$*/UGP*)]/2	−0.476	0.082
	(0.058)	(0.047)
α_{j2} ln(own price/*RPI*)	−0.185	−0.014
	(0.055)	(0.045)
$\alpha_{j3}\Delta$ ln(own price/*RPI*)	−0.097	−0.100
	(0.045)	(0.042)
α_{j4} ln(own price/*BRP*$_{-1}$)	−0.082	−0.160
	(0.054)	(0.045)
α_{j5} ln(*BSP/UGP*)$^{a}_{-13}$	−0.246	0.106
	(0.059)	(0.049)
α_{j6} ln(own price/*BRP*)$^{a}_{-13}$	−0.073	–
	(0.040)	
α_{j7} ln(*BRP/RPI*)$^{a}_{-13}$	–	−0.049
		(0.062)
δ_{j1} *TREND*	−0.035	−0.010
	(0.004)	(0.002)
δ_{j2} *RAIN*	–	0.038
		(0.034)

TABLE 2 (continued)

		ln(*BSQ/POP*)	ln(*UGQ/POP*)
δ_{j3} *TEMP*/100		0.051	–
		(0.019)	
δ_{j4} *JAN*		–	−0.090
			(0.016)
δ_{j5} *JAN*TEMP*/100		–	−0.126
			(0.066)
δ_{j6} ln*SEEMP*		–	1.300
			(0.185)
$\delta_{j7}\Delta_{13}$ln*SEEMP*		1.006	–
		(0.149)	
δ_{j8} ln*VIS*		–	0.081
			(0.021)
s.e.	Standard error	0.034	0.028
DW	Durbin-Watson statistic	1.876	1.794
LM_{13}	Lagrange Multiplier	2.663	2.241
LM_4	Serial correlation test	0.128	0.210
df	Degrees of freedom	176	175
	Dependent mean	6.060	5.839
	Variable s.d.	0.343	0.121

Sample: 1972:1 to 1987:10; asymptotic standard errors in parentheses; constant and seasonal dummy coefficients omitted.

implies that a rise in bus fares, with underground and British Rail fares held constant, would result in a fall in revenue. These estimates indicate a very substantial degree of potential competition between the bus and underground networks. In particular, revenues from the bus network will depend in a sensitive manner on underground prices.

For comparison, we also give in Tables 4a and 4b the same elasticities with the Slutsky symmetry condition (5) imposed on the long-run responses. The effect is to increase very substantially the estimated long-run price responsiveness of demand for underground travel. It does therefore seem clear that the aggregate time series data are incompatible with the standard integrability condition of demand theory, and that imposition of this restriction will tend to exaggerate the

TABLE 3a

Estimated Impact Price Elasticities

	Prices			
	Bus	*Underground*	*British Rail*	*Non-travel*
Bus	−0.839	0.476	0.082	0.281
Underground	0.041	−0.355	0.160	0.114

TABLE 3b

Estimated Long-Run Price Elasticities

	Prices			
	Bus	*Underground*	*British Rail*	*Non-travel*
Bus	−1.318	0.897	0.193	0.229
Underground	0.356	−0.688	0.211	0.120

price responsiveness of underground travel.[9]

The rejection of Slutsky symmetry may be attributed to omission in the model of the price of travel by private (automobile) transport. In London, underground travel is very largely transport from Outer London to Central London, while bus travel, though partly also fulfilling this function, provides the main public transport facilities in Outer London. It is therefore reasonable to suppose that the private car is a much closer substitute for bus travel than for underground travel. If this is so, it follows that omission of the price of private transport is likely to

[9] Bös (1986) estimated an Almost Ideal (AI) demand system (Deaton and Muellbauer, 1980) on annual data, distinguishing bus travel, underground travel, private traffic and other consumption. He found own-price elasticities of −0.80 and −0.98 for bus and underground respectively, and a cross-price elasticity of 0.25. His estimated own elasticities are comparable with our Table 4b estimates. Unfortunately, Bös does not report a test of the AI specification relative to an unrestricted model. Our suspicion is that his high estimated underground elasticity is the outcome of this untested restriction.

TABLE 4a

Estimated Impact Price Elasticities with Long-Run Symmetry Imposed

	Prices			
	Bus	*Underground*	*British Rail*	*Non-travel*
Bus	−0.788	0.414	0.096	0.278
Underground	0.078	−0.396	0.182	0.058

TABLE 4b

Estimated Long-Run Price Elasticities with Long-Run Symmetry Imposed

	Prices			
	Bus	*Underground*	*British Rail*	*Non-travel*
Bus	−1.185	0.724	0.240	0.221
Underground	0.661	−0.983	0.166	0.156

bias the estimated own and cross bus price elasticities by more than the corresponding underground elasticities (Gilbert and Jalilian, 1989).

Finally in this section we report the estimated version of the bus miles run (*BSMR*) equation (16). This is

$$\begin{aligned}\ln BSMR_t = {} & \underset{(0.057)}{0.604}\ln BSMR_{t-1} + \underset{(0.036)}{0.088}\,\Delta \ln BSQ_{t-1} \\ & + \underset{(0.028)}{0.043}\,\Delta \ln (BSP_t/RPI_t) + \underset{(0.011)}{0.014}\ln (BSP_{t-1}/AVER_{t-1}) \\ & - \underset{(0.004)}{0.034}\,(BHOLS_t - \underset{(0.057)}{0.604}\,BHOLS_{t-1})\end{aligned} \tag{20}$$

s.e. = 0.026 DW = 2.126 df = 186
Lagrange multiplier test: LM (13,173) = 2.381; LM (4,182) = 0.279
Dependent variable: mean = 9.487, sd = 0.060.

4. THE FARECARD MODEL

We report the estimated equations for the quantum of underground season tickets (*UGSQ*), the quantum of bus passes (*BSSQ*), the number of Travelcards (*TCDS*) and the number of Capitalcards (*CAPS*). A general feature of the data for all four cards is the marked and not obviously constant seasonality. For many travellers, the decision whether or not to purchase a farecard is quite finely balanced, and the expectation of missing one or more days' work because, for example, of a bank holiday may show a cost saving from travelling on an ordinary basis for a week or a month. It is therefore to be expected that farecard validity will show more marked seasonality than the underlying travel decision, and in fact it does.

A second complication is that the different farecards have been marketed for different periods. This prevents us from using a systems estimator jointly for the travel quanta and for the four farecard variables, at least in any simple way. For the same reason it is difficult to estimate demand for the different farecards jointly. We have therefore decided to adopt a single-equation approach to this section of the model. This approach has the virtue of allowing flexibility in functional specification.

In each case we attempt to relate the farecard validity variable to the overall travel quantum for the relevant model, defined as the travel quantum that would have been observed on that mode in the absence of any farecards. We can construct these travel quanta, which we write as BSQ^* and UGQ^* for the bus and underground networks respectively, by using the estimated displacement coefficients (the θs) from Table 2. This gives[10]

$$BSQ_t^* = BSQ_t + 33.41\,BSSQ_t - 5.03\,TCDS_t + 17.37\,CAPS_t \qquad (21b)$$

and

$$UGQ_t^* = UGQ_t + 10.54\,UGSQ_t + 15.89\,TCDS_t. \qquad (21u)$$

Capitalcard validity is related to British Rail travel by means of a series *PKABR*, derived by interpolating on to a four-weekly basis the annual cordon census data on British Rail peak arrivals at Central London termini.

It is natural to suppose an equilibrium relationship between farecard validities and the underlying travel quanta, and it is reasonable to suppose that these relationships exhibit a unit elasticity. That implies that if the underlying quantum (say the underground quantum) rises by 1 per cent then, in the long run, underground season ticket sales will also rise by 1 per cent. However, the short-run response may be different from this (and usually less than the long-run response). This view may be modelled by adopting the "error correction" specification popularised by Davidson *et al.* (1978) and more recently rationalised in terms of cointegrated processes by Engle and Granger (1987).[11] Thus, if n_{it} is the

[10] As noted, the coefficient of *CAPS* is set to zero until 1985:7.

[11] See Nickell (1985) and Gilbert (1986) for introductory discussions.

farecard validity of card type i and y_{jt}^* is the relevant travel quantum variable, we consider models of the form

$$\Delta \ln n_{it} = \alpha_0 + \alpha_1 \Delta \ln y_{jt}^* - \alpha_2 (\ln n_{i,t-1} - \ln y_{j,t-1}^*) \tag{22}$$

Equation (22) implies a long-run farecard to travel quantum ratio of[12]

$$\frac{n_i}{y_j^*} = e^{\alpha_0/\alpha_2} \tag{23}$$

We adopt the error correction specification for all four farecard equations.

The underlying principle of the farecard model is that farecard validity may be related to the travel quantum on the relevant mode. This view implies that demand shift variables, such as our measure of employment in the South East, should only affect farecard validity through the underlying travel quantum; and this implies testable restrictions. Specifically, we test the hypothesis that the demand shift variables in the underground quantum equation in Table 2, ln*SEEMP* and ln*VIS*, have zero coefficients in the underground season tickets equation, and that the single demand shift variable Δln*SEEMP* in the bus quantum equation has a zero coefficient in the two-pass equation. These tests correspond to (d) in Section 1.

These tests are complicated by the fact that quantum variables *BSQ** and *UGQ** are clearly jointly determined with farecard validities, and indeed are constructed with these variables as components (21b,u). Estimation of the farecard equations is therefore by Instrumental Variables (IV), and in the underground season ticket and Travelcard equations the relevant tests statistics, which have only asymptotic validity, are Likelihood Ratio (LR) statistics.

Underground season tickets were available from the start of the sample till 1983:5. The most satisfactory equation was

$$\begin{aligned}\Delta \ln UGSQ_t - \Delta \ln UGQ_t^* = &-0.966 \Delta \ln(UGSP/RPI)_t - 1.116 \Delta \ln(UGP/RPI)_t \\ &(0.370) \qquad (0.364) \\ &+ 0.131 \ln(BRP/RPI)_t - 0.350 \ln(UGSP/UGP)_{t-1} - 0.163\, DUM827 \\ &(0.138) \qquad (0.106) \qquad (0.054) \\ &+ 0.122 \ln UGQ_t^* - 0.341 \ln UGSQ_{t-1} + 0.125 \ln UGSQ_{t-6} \\ &(0.057) \qquad (0.052) \qquad (0.039) \\ &+ \text{constant} + \text{seasonals}\end{aligned} \tag{24}$$

IV, sample: 1972:1 to 1983:5; s.e. = 0.046 DW = 2.437
Dependent variable: mean = 0.002 s.d. = 0.115
Serial correlation: LM(13): χ^2_{13} = 26.52 (95% c.v. = 22.4)
Endogenous regressor: lnUGQ_t^*
Additional instruments: ln(*SEEMP*), *TREND, JAN, JAN*TEMP*
Instrument validity: χ^2_3 = 0.87 (95% c.v. = 7.81)
LR test for exclusion of ln*SEEMP* and ln*VIS*: χ^2_2 = 1.57 (95% c.v. = 5.99)

[12] Setting the growth rate $\Delta \ln y^* = 0$ for simplicity.

The exclusion restrictions are easily satisfied. There is evidence of serial correlation, but no very clear pattern is evident, and it is likely that this represents non-constant seasonality. The implied steady state solution is

$$\ln UGSQ = 0.562 \ln UGQ^* - 1.613 \ln(UGSP/UGP) \qquad (24e)$$

Note the very high implied equilibrium price elasticity of −1.6 with respect to the relative price of season to ordinary tickets.

Bus passes have been available throughout the sample we have used for estimation of the ordinary quanta equation. However, there is evidence of considerable structural instability in the relationship, which may reflect differing conditions of validity. Since we are not primarily interested in explaining take-up of bus passes, we avoided this problem by estimating an equation over a relatively short period starting in the autumn of 1980. The estimated bus pass equation is

$$\begin{aligned}\Delta\ln BSSQ_t = {} & \underset{(0.408)}{1.955}\,\Delta\ln BSQ^*_t + \underset{(0.028)}{0.134}\,TREND - \underset{(0.167)}{0.163}\,\ln(BSSP/BSP)_t \\ & - \underset{(0.073)}{0.350}\,\ln BSSQ_{t-1} - \underset{(0.086)}{0.180}\,\ln BSSQ_{t-3} + \underset{(0.210)}{0.264}\,\ln BSQ^*_{t-1} \\ & - \underset{(0.182)}{0.722}\,\ln UGQ^*_{t-1} - \underset{(0.015)}{0.051}\,\ln TCDS_{t-3} + \underset{(0.126)}{0.849}\,DUM824 \\ & - \underset{(0.095)}{0.660}\,DUM836 - \underset{(0.082)}{0.174}\,DUM8559 + \text{constant} + \text{seasonals}\end{aligned} \qquad (25)$$

IV, sample 1980:10 – 1987:10; s.e. = 0.084, DW = 2.049
Dependent variable, mean = 0.010, s.d. = 0.157
Serial correlation, $\chi^2_6 = 6.72$ (95% c.v. = 12.59)
Endogenous regressor: $\Delta\ln BSQ^*$
Additional instruments: $\ln SEEMP$, $\ln(BSP/BRP_{-1})$, $\ln(BSP/UGP)$, $\ln(BSP^a/BRP^a)_{-13}$, $\ln(BSP^a/UGP^a)_{-13}$, $\Delta\ln(BSP/RPI)$, $\Delta_{13}\ln SEEMP$
Instrument validity: $\chi^2_7 = 15.54$ (95% c.v. = 14.07)
LR test for exclusion of $\ln SEEMP$: $t_{63} = 1.45$

The dummy variable for 1982:4 indicates a large increase in validity of bus passes, possibly associated with changes in the validity basis during that period; the 1983:6 dummy relates to the introduction of Travelcards; and the dummy *DUM*8559 relates to the introduction of Capitalcards in 1985. The implied long-run solution is

$$\begin{aligned}\ln BSSQ = {} & 0.498\,\ln BSQ^* - 1.361\,\ln UGQ^* - 0.094\,\ln TCDS \\ & - 0.308\,\ln(BSS/BSP) + 0.253\,TREND\end{aligned} \qquad (25e)$$

This shows a modest elasticity with respect to bus travel, offset by a very much larger negative elasticity with respect to underground travel, and indicates a long-term loss in market share to the underground network. We were unable to find any long-term effect from the introduction of Capitalcards, except on the overall bus quantum *BSQ**. The price elasticity at −0.3 is relatively low in comparison with that for underground season tickets.

In the light of the estimates reported in Table 2, it makes sense to relate

Travelcard validity to the overall underground quantum UGQ^*. Our estimated equation is

$$\begin{aligned}\Delta \ln TCDS_t = {} & \underset{(0.129)}{0.686}\, \Delta \ln UGQ^*_t - \underset{(0.141)}{0.551}\, \ln(TCDP_{t-3}/UGP_t) \\ & - \underset{(0.095)}{0.524}\, \ln(TCDS/UGQ^*)_{t-1} + \underset{(0.044)}{0.252}\, \ln(TCDS/UGQ^*)_{t-13} \\ & + \text{constant} + \text{seasonals}^{13}\end{aligned} \tag{26}$$

IV, sample: 1984:6 – 1987:10; s.e. = 0.021, DW = 1.720
Dependent variable: mean = 0.007, s.d. = 0.090
Serial correlation: LM(2) = 3.54 (95% c.v. = 5.99)
Endogenous regressor: $\Delta \ln UGQ^*_t$
Additional instruments: full set of seasonals, trend, $\ln SEEMP$, JAN^*TEMP, $\ln VIS$, $\ln(UGP/BRP_{-1})$, $\ln TCDS_{-1}$, $\ln TCDS_{-13}$, $\ln(BRP^a/RPI^a)_{-13}$, $\ln(BSP^a/UGP^a)_{-13}$, $\Delta \ln(UGP/RPI)$, $[\ln(BSP/UGP) + \ln(BSP_{-1}/UGP)]/2$
Instrument validity: $\chi^2_{13} = 12.78$ (95% c.v. = 22.4)
LR test for exclusion of $\ln SEEMP$ and $\ln VIS$: $\chi^2_2 = 2.86$ (95% c.v. = 5.99)

This equation imposes a long-run unit elasticity with respect to the total underground quantum. A χ^2_2 likelihood ratio test of this restriction gives a value of 0.03, which is clearly acceptable. Again, the exclusion restrictions are satisfied. As in the bus pass equation, the relative price term has an anticipatory interpretation: an announced rise in ordinary ticket prices results in increased purchases of longer-duration Travelcards as travellers lock into the currently lower prices, and this increases validity over the following months. The implied long-run solution is

$$\ln(TCDS/UGQ^*) = -2.026\, \ln(TCDP/UGP) \tag{26e}$$

which shows a very high elasticity with respect to ordinary ticket prices.

Capitalcards were introduced at the beginning of 1985, and over our sample, which finishes in late 1987, the data do not show much apart from a positive trend overlaid with seasonal influences. It was apparent in the modelling of Capitalcard validity that some non-linearity was present, and this suggested use of the logistic transform.

$$\text{lgt}(CAPS_t/PKABR_t) = \ln\left(\frac{PKABR_t - CAPS_t}{CAPS_t}\right) \tag{27}$$

This transformation has in any case the desirable implication that penetration is contained within the unit interval, so that the number of Capitalcards is bounded above by British Rail peak arrivals. The estimated equation, which again has the error correction form, is

[13] Considerations of degrees of freedom led us to use a reduced set of seasonal dummies.

$$\Delta\text{lgt}(CAPS/PKABR)_t = \underset{(0.0117)}{-0.0253} - \underset{(0.0235)}{0.0347}\ \text{lgt}(CAPS/PKABR)_{t-1} + \text{seasonals} \quad (28)$$

OLS, sample 1985:2 – 1987:10; $R^2 = 0.563$, s.e. = 0.057, DW = 2.29
Dependent variable: mean = –0.0407, s.d. = 0.0815
Serial correlation: LM(3) = 2.48

We were not able to determine any price effects on Capitalcards, but this probably only reflects the lack of any substantial movement in the relevant relative prices over this short sample. A notable feature of equation (28) is the absence of any time trend, the upward trend in Capitalcards being captured entirely through the partial adjustment mechanism. Solution of (28) indicates that, if the January 1989 rationalisation of Travelcards and Capitalcards had not taken place, Capitalcard validity would have settled down at around 67.5 per cent of British Rail peak arrivals. Given the short sample employed to estimate this equation, there seemed to be little point in attempting to test variable exclusion restrictions.

5. SIMULATED EFFECTS OF INTRODUCTION OF TRAVELCARDS

We use the joint ordinary ticket and farecard model to attempt to estimate the travel and revenue effects on LRT of the introduction of Travelcards in April 1983 and of Capitalcards in January 1985. The initial step in this exercise was to generate a base simulation starting in 1983:6 (the date at which Travelcards were introduced), with which the subsequent counterfactual simulations can be compared.[14] The simulation performance of the model over this period is summarised in Table 5.

The counterfactual simulations were performed by (i) rerunning the model, dropping both the Travelcard and Capitalcard equations, but with the underground season ticket equation reactivated;[15] and (ii) rerunning the model with only the Capitalcard equation dropped. Comparison of (ii) with the base simulation allows inference about the introduction of Capitalcards; comparison of (i) with the base simulation, netting out the effects of (ii), allows inference about the introduction of Travelcards. The results of these two sets of simulations are summarised in Table 6a (effects of Travelcards) and Table 6b (effects of Capitalcards).

There are no problems in estimating the effects of these developments on the bus and underground ordinary ticket quanta, reported in the first and fourth lines respectively of Tables 6a and 6b. As noted in Section 2, we do not have trip data, and therefore cannot directly simulate these quantities. We might in principle use the estimated θ coefficients to augment the ordinary ticket quanta to take into account travel covered by farecards, but the estimated displacement effects reported in Table 2 seem too low to be plausibly regarded as travel

[14] We used deterministic simulations, despite the nonlinearity of the model.

[15] Recall that underground season tickets were no longer sold from 1985:6, so this equation is absent from the base simulation.

TABLE 5

*Simulation Errors**

		Bias	*Standard Deviation*	*Root Mean Square Error*
Bus ordinary quantum	*BSQ*	1.4	7.1	7.2
Underground ord. quantum	*UGQ*	1.5	4.7	4.9
Quantum of bus passes	*BSSQ*	1.7	23.4	23.4
Number of Travelcards	*TCDS*	1.2	17.6	17.6
Number of Capitalcards	*CAPS*	2.2	24.1	24.2
Bus miles run	*BSMR*	2.4	2.8	3.7

* As percentage of simulation mean.
Simulation period: 1983:6 → 1987:10 except for Capitalcards (1985:1 → 1987:10).

propensities. We therefore prefer to use numbers which might represent the travel intensities of the representative farecard holder. For our four-weekly periods we set $\theta_{b2} = \theta_{u1} = \theta_{u3} = 40$, $\theta_{b3} = 8$, and $\theta_{b4} = \theta_{u4} = 20$. The implied bus and underground quanta are written as BSQ^{**} and UGQ^{**} respectively. It should be emphasised that these numbers, though plausible, are quite conjectural, and that considerable caution should therefore be exercised in interpreting BSQ^{**} and UGQ^{**} as measures of the number of trips undertaken. The revenue calculations reported in Tables 6a and 6b are more reliable, but we experienced difficulty in obtaining a satisfactory measure of the effective price of Capitalcards (the price depends on the station of origin),[16] and we have not been able to take into account the growth in LRT revenues resulting from increased sales of LRT tickets (both ordinaries and farecards) by British Rail. As a percentage of underground receipts, this category grew from around 7 per cent in 1983 to around 15 per cent by the end of 1987.

The Travelcard simulations, reported in Table 6a, show the expected substantial displacement of underground ordinary fares (around 19 per cent of the total in 1986 and 1987), together with a modest trip generation effect on bus ordinary fares (around 8 per cent). These imply an increase of 10 per cent in underground trips, but a more substantial increase (around 16 per cent) in bus trips. The increase in underground trips arises from the fact that Travelcards are more popular with potential underground travellers than were underground season tickets, and so as a marketing device they may be judged successful. The more

[16] The revenue simulations reported here use a constant (four-weekly) Capitalcard price of £2.26 divided between the underground and bus networks in the ratio 0.738:0.262. This price was obtained as an average of London Underground receipts from Capitalcards divided by card validity scaled to reflect the underground-bus apportionment; the ratio reflects the apportionment ratio.

substantial increase in bus trips follows from our estimated positive θ_{b3} coefficient (Table 2) that Travelcards have generated additional paid bus trips, together with our reasonable assumption that Travelcard holders also make a few unpaid bus trips that they would not otherwise have undertaken. The generation of paid trips may arise from accompanying persons (for example, children accompanying a parent with a Travelcard), as suggested by Jones (1987); or perhaps bus travel in a period in which one holds a card may, through increased familiarity with the bus network, induce travel in a period in which one no longer has a card.

The simulation results suggest that Travelcards have boosted bus revenues by around 14 per cent (relative to the base simulation), and that underground revenues are down by almost the same amount. However, this may be the result of our failure to model all flows of LRT revenue, particularly the increased number of tickets sold by BR. The rise in bus revenues follows from the fact that London Buses both receives 30 per cent of Travelcard revenues and also has enjoyed a greater number of fare-paying passengers, though the introduction of Travelcards has involved some decline in bus pass validity. London Underground, on the other hand, has substituted Travelcards, from which it only receives a 70 per cent apportionment, for underground season tickets, which gave a 100 per cent apportionment.

The simulated effect of the 1985 introduction of Capitalcards is quantified in Table 6b. Bus ordinary travel is estimated to fall by around 10 per cent, though the estimated number of bus trips is seen to rise. There is no effect on underground ordinary ticket sales. This reflects our finding (Table 2) that the displacement effect of Capitalcards was entirely in terms of bus ordinaries. As a result of the revenue apportionment underground revenues rise by around 2.5 per cent while bus revenues fall by up to 8 per cent.

6. CONCLUSIONS

Our approach has been to model the decision to purchase a farecard as separable from the decision to travel, and as depending on the amount of travel that would have been undertaken in the absence of farecards. Implementation of this approach for the LRT network is hampered by the lack of trip data. However, we showed in Section 2 that the hypothesis implies a relatively simple simultaneous model relating ordinary ticket and farecard purchases. We have implemented that model in this paper, and its performance seems relatively satisfactory.

The modelling framework implies two sets of tests. The simultaneous structure entails a set of intra-equation restrictions relating the dynamic adjustment of ordinary ticket purchases to their own lagged values and to lagged farecard validities. We tested these restrictions in section 3 and concluded that they are broadly satisfied, though there is a problem in relating to the identification of the contemporaneous effects of changes in farecard validity. The second set of tests suggested by the model is the exclusion restriction on the determinants of the overall travel quanta in the farecard equations. We tested these restrictions in section 4 and found them broadly satisfied.

TABLE 6a

Estimated Impact of Introduction of Travelcards

	% increases relative to base simulation				
	1983	*1984*	*1985*	*1986*	*1987**
Bus ordinary quantum (*BSQ*)	2.7	6.1	7.5	8.1	8.7
Overall bus quantum (*BSQ***)	6.7	14.8	15.7	16.2	16.9
Total bus revenue	4.1	9.9	12.2	13.2	14.6
Underground ordinary quantum (*UGQ*)	−4.7	−12.5	−16.2	−18.3	−19.5
Overall underground quantum (*UGQ***)	−7.3	6.3	9.0	9.7	10.1
Total underground revenue	−7.5	−10.2	−11.5	−12.3	−12.8
Total LRT revenue	−1.9	−0.5	−0.6	−0.9	−1.5

* January to October.

TABLE 6b

Estimated Impact of Introduction of Capitalcards

	% increases relative to base simulation				
	1983	*1984*	*1985*	*1986*	*1987**
Bus ordinary quantum (*BSQ*)	–	–	−3.9	−5.5	−11.2
Overall bus quantum (*BSQ***)	–	–	5.3	5.3	6.7
Total bus revenue	–	–	−2.7	−3.7	−8.7
Underground ordinary quantum (*UGQ*)	–	–	–	–	–
Overall underground quantum (*UGQ***)	–	–	5.7	7.7	8.8
Total underground revenue	–	–	1.8	2.4	2.7
Total LRT revenue	–	–	−0.3	−0.3	−1.8

* January to October.

Overall, therefore, our approach to the joint modelling of travel demand and demand for farecards appears to be justified. At the same time we recognise that it would be tested more stringently if the model could be implemented in the context of a travel authority which has continuous trip data available. We hope this study may stimulate research along these lines.

A third set of restrictions that we investigated was the Slutsky symmetry conditions on the cross price effects in the bus and underground demand functions. These are decisively rejected. We conjecture that the reason for the

rejection is lack of information on private automobile transport. This does suggest that if one wishes to obtain reliable information on travel demand elasticities one will need to use detailed survey data in conjunction with time series data.

Our estimated long-run cross price elasticities (Table 3b) show a high (0.9) cross elasticity of demand for bus services with respect to the underground price, but much lower elasticities for underground services relative to the bus price (0.35) and for both bus and underground services relative to British Rail prices (0.2). Overall, we find that underground services are inelastically demanded, while bus services are elastically demanded. The price implications of these estimates depend upon LRT's objectives, but the figures do suggest that London Underground is in a significantly stronger competitive position than London Buses.

There are two ways in which Travelcards may generate additional travel. First, a greater proportion of the travelling public may use the LRT network; and, second, Travelcard holders may make additional trips. It is difficult to obtain evidence of the second effect from the data we have available. However, our results indicate that Travelcards appear to have generated additional paid bus travel, rather than displacing ordinary travel as on the underground network. This may be because additional travellers accompany persons with Travelcards, or because Travelcard purchase familiarises potential travellers with the bus network, and they subsequently use it for paid travel.

The results of simulating the revenue implications of the introduction of Travelcards and Capitalcards are quite dramatic. The revenue from Travelcards was, before January 1989, apportioned in the ratio 70:30 between London Underground and London Buses. However, we have already noted that London Buses have actually gained in terms of paid (ordinary) fares from Travelcards. For London Underground, on the other hand, Travelcards replaced underground season tickets, from which they received the entire revenue. Our simulation results therefore suggest that the introduction of Travelcards may have resulted in a significant shift of revenue from London Underground to London Buses. The situation is however complicated by consideration of Capitalcards; their apportionment appears to have been more favourable to London Underground. But these simulation results are conditional on the validity of our model, and the problems encountered in obtaining an adequate time series for employment in Central London suggest that they should be treated with caution.

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