THE TRAVEL TO SHOP BEHAVIOUR OF CONSUMERS IN EQUILIBRIUM MARKET AREAS

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One of the major components of road travel is that generated by shopping trips. For example, Vickerman and Barmby (1984) cite UK evidence that 17 per cent of all travel journeys (excluding walking trips of under one mile) were for shopping, while the journey to work accounted for 26 per cent of all trips. There are many empirical studies of shopping journeys but there is little analysis based on behavioural models of the consumer. In order to understand the characteristics of journeys to shop, we must identify the forces which lead to the choice of one shop rather than another, and also the factors which determine the frequency at which that shop is visited. Conventional models of spatial consumer behaviour (see Greenhut, Norman and Hung (1987) for a recent account) have concentrated on the amounts purchased by individual consumers at their chosen shopping centre, without analysing the frequency at which the centres are used. However, a key feature of such models is that they have shown how to identify the market areas of the competing centres: that is, the set of consumers who choose to use a particular centre. This generates one dimension of the total travel to use that centre, but without evidence on frequency a complete picture cannot be given. When the models of consumer behaviour are adapted to allow for the frequency of shopping, whether this is exogenous or endogenous, a complete picture can be derived of the number of trips per period from each distance to the shop.

A recent paper by Bacon (1989) has integrated earlier studies of the frequency of shopping with the traditional spatial competition models of market area by authors such as Capozza and Van Order (1978). This approach makes the frequency of shopping, and hence the resulting equilibrium market areas, a function of several economic variables (for example, income and various transport costs). The present paper, building on this model of equilibrium consumer shopping, derives the average travel to shop behaviour over the set of consumers using a given shopping centre. The number of trips per period made by all consumers using the centre, the average number of trips per period over all consumers, the average trip length over all consumers and the average total distance travelled to shop per period, are all obtained from this modelling of shopping. Using these expressions, the effects of parameter variations on travel characteristics are then explored. These parameter variations have a twofold effect on the total travel to shop:

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(i) for a market of given size changes in income, and so on, affect the behaviour of individual consumers;
(ii) as individual shopping behaviour alters, the competitive equilibrium market size reacts; thus consumers are added (or removed) from the market for a given shop and so travel characteristics must be averaged over a different group of consumers.

The model developed is too complex, even though it begins with very simple functional forms, to permit simple closed analytical expressions for the derivatives of the various travel characteristics. So instead the effect of each parameter shift is calculated by numerical methods for certain values of the other parameters of the model. This allows us to evaluate the relative importance of the various determinants of the travel to shop.

A MODEL OF THE FREQUENCY OF SHOPPING

Traditional models of spatial economic behaviour, such as that developed by Capozza and Van Order (1978) and extended by Greenhut, Norman and Hung (1987) consider the problem of finding the equilibrium market size served by shopping centres in a town, and the price charged. This is done by modelling supply and demand and solving for the number of centres that leaves each shop making zero profits while marginal profits are also zero (so that each shop is at its profit maximising position). The key to these models is the demand function, which, for a consumer living t units distance away from a shopping centre, has conventionally been assumed to be of the form:

$$Q = a - b(p + gt) \tag{1}$$

where

Q	is quantity demanded,
p	is the "mill" price of the good,
g	is the transport cost per unit distance,
t	is the distance travelled to shop,
a and b	are demand parameters.

This standard formulation implies that transport costs are defined as being per unit purchased (per unit distance carried). This assumption, that the transport cost of shopping is proportional to *the amount purchased on the trip,* appears highly unrealistic in relation to consumer behaviour. The formulation (1) also implies that the frequency of shopping (which is not explicitly introduced into traditional models) is indeterminate — that shopping twice as often to buy half as much per trip leaves total transport (and purchase) costs per time period unchanged. In order to resolve this indeterminacy, while making a more plausible assumption about the nature of travel costs, it is assumed firstly that the cost of travel per trip does not depend on the amount transported. It is assumed that there are two elements to the travel cost — one is a fixed cost element, irrespective of the distance travelled, and the second element is a cost proportional solely to the distance travelled. Models of this kind have been proposed by Reinhardt (1973) and Bacon (1984). The cost of a single trip is thus $gt + w$, where g is independent of the quantity and w is the

fixed cost per trip. The part of travel costs proportional to distance is the time and money involved in the travel itself, while the fixed cost element is the time taken to prepare for shopping and for the actual shopping process. Once the cost of shopping trips is made independent of the quantity purchased per trip, then, in the absence of other costs, it would be optimal to make as few trips as possible and buy a very large quantity on each of these infrequent trips. This implausibility is countered by recognising the presence of inventory costs. These costs are the costs of perishability, the opportunity costs of the money involved, and actual physical inventory costs. Such costs penalise over-large bundles of shopping. If the consumer uses up purchases at a uniform rate and goes shopping as soon as the inventory is empty, and if there is a cost of $2c$ units per period for holding 1 unit of value in the inventory for one period, then the consumer's choice problem can be stated: Choose q (bundle size) and f (frequency) so as to maximise the per period consumption (utility) $Q(= qf)$, subject to the budget constraint:

$$Y = pqf + cpq + (tg + w)f \tag{2}$$

Here Y is disposable income per period. The optimal values are:

$$q^* = \{-(gt + w) + [(gt + w)^2 + (gt + w)\ Y/c]^{0.5}\}/p \tag{3}$$

$$f^* = \{-c + [c^2 + cY/(gt + w)]^{0.5}\} \tag{4}$$

These equations are discussed in detail by Bacon (1989), but for this analysis of the travel characteristics of shopping we focus on certain aspects of optimal frequency:

(1) Frequency depends on economic factors such as income, variable and fixed travel costs, and inventory costs. A change in any of these will alter the frequency of shopping for each individual.

(2) Frequency depends on distance to the shops in a non-linear fashion. The lower bound is zero (at extreme distances), so that all consumers in the market will use the shop — no consumers are excluded because of distance, as some can be with a demand function linear in distance.

(3) The frequency of shopping is greatest for consumers living next to the shopping centre. The finite upper bound on frequency is crucially dependent on the fixed cost of shopping w.

The simple model of endogenous frequency of shopping generates the number of trips per period made by each consumer in the market. In order to obtain averages over all consumers, it is necessary to model the population density of households within the market and to identify the limits of the market for an individual shopping centre (that is, which households use a given centre). Virtually all work on urban modelling takes the density of households as uniform; we follow this by assuming a linear town with uniform population density D per unit length.

Identification of the market area of a shopping centre requires us to specify the costs and competitive behaviour of rival shops. Following earlier analysis, we assume that:

(i) each shop has a total fixed cost per unit time period a;

(ii) each shop has a constant variable cost of b per unit sold;

(iii) each firm, in deciding on its location and price, assumes that its rivals will not react to its decisions;

(iv) each firm acts so as to maximise its expected profits;

(v) firms enter or leave the market until economic profits are everywhere zero.

Under these assumptions, total profits per shop are set equal to zero for a market boundary T (the line half way between rival shops) which satisfies the equation:

$$\Pi(p,T)=2(p-b)\,D/p\int_0^T E(t)\,dt-a=0 \tag{7}$$

where $E(t)$ is the expenditure per period on the good by a consumer located t units from the shop. Profits are maximised when:

$$\frac{d\Pi}{dp(T)}=0 \tag{8}$$

Analysis of the solutions for T and p is made more complex by the fact that the market boundary between the two shops changes if one shop alters its price.

It can be shown that the profit maximising equation yields:

$$(p-b)\,[E(T)]^2+2be(T)E'(T)=0 \tag{9}$$

where $e(T)$ is the integral of $E(t)$ evaluated at the upper limit of T, and $E'(T)$ is the derivative of the expenditure function with respect to distance t evaluated at T. This, together with the zero profit equation which can be written as

$$2(p-b)\,De(T)/p-a=0 \tag{10}$$

can be solved to eliminate p:

$$aE^2+2eE'\,(2eD-a)=0 \tag{11}$$

which is an implicit function of T. Given the very complicated nature of the integral, it is not possible to obtain a closed expression for the solution of T in terms of the parameters of the model. However, the solution of (11) for given parameter values can be numerically approximated, so that we can calculate the size of the market area over which the characteristics of the travel to shop are to be analysed.

TRAVEL CHARACTERISTICS

In the first part of the paper we have shown that a consumer living t units from a shopping centre will make $f(t)$ shopping trips per unit time. Since we can estimate the market boundary for a given set of parameter values, we can derive average travel behaviour for the set of consumers using that shopping centre. Further, since in an equilibrium situation all shopping centres attract identical market areas, this will also be the average for all consumers living in the town.

Given the uniform density of consumers (D), the total number of shopping trips made to the centre per period of time from one direction is:

$$N=D\int_0^T f(t)\,dt \tag{12}$$

The average number of trips to shop per period per shopper is:

$$AN = D\int f(t)\,dt / D\int dt \tag{13}$$

$$= \int f(t)\,dt / T = N/DT \tag{14}$$

The average distance travelled to shop per unit time per person is:

$$AD = D\int t f(t)\,dt / D\int dt \tag{15}$$

and the average journey length per person is:

$$AJ = D\int t f(t)\,dt / D\int f(t)\,dt \tag{16}$$

$$= AD/AN \tag{17}$$

To evaluate these expressions we need to evaluate the two integrals, given the expression for $f(t)$ — these are shown in the Appendix. Using these integrals and a set of parameter values, together with the calculated values of the market boundaries T, we can derive the travel characteristics N, AN, AD and AJ.

PARAMETER SHIFTS AND THE TRAVEL TO SHOP

The analysis of the effects of parameter shifts on the various aspects of average travel to shop can be broken into two parts. In the first part we consider a fixed market size — we allow for the fact that shifts in certain of the parameters alter the behaviour of those consumers in the market area of a shop, but we do not allow for the fact that these changes will also lead to an adjustment of the market boundaries and therefore to a change in the number of consumers who use a given shop. This distinction corresponds to the short-run/long-run analysis of the theory of the firm. In the second stage of the analysis we account for the changes in market area that result from the changing consumer behaviour, so that the averages are taken over different numbers of consumers. The short-run analysis relates solely to shifts in demand parameters, since the cost parameters enter the model only by their effect on equilibrium market area (that is, they do not affect the behaviour of individual consumers).

The key to the analysis is the behaviour of the individual shopper as given by (4). For a consumer located t units away from the shopping centre we have the following results for the signs of the partial derivatives:

$$f_y > 0; f_c > 0; f_g < 0; f_w < 0 \tag{18}$$

Comparing consumers at different distances from the centre, with all other parameters held constant, we also have:

$$f_t < 0. \tag{19}$$

The density parameter (D) enters as a general scaling factor for a market of a given size, so that we can immediately derive the qualitative results shown in Table 1 for the effects of a shift in the parameter of interest (labelled k) in a fixed market area.

TABLE 1

Effects of Parameter Shifts on Short-run Travel Characteristics

k	N_k	AN_k	AD_k	AJ_k
Y	+	+	+	?
c	+	+	+	?
g	−	−	−	?
w	−	−	−	?
D	+	0	0	0
T	+	−	+	+

The signs for the directions of the impacts of the parameter shifts are obtained by elementary reasoning. For example, when income rises each consumer in the given market area makes more trips to the centre. As a result the total number of trips to the centre (N), the average number of trips per period per person (AN) and the average total distance travelled to shop per period per person (AD) all increase. The average journey length (AJ) is indeterminate, because it depends on whether the incremental effect on frequency is greater for consumers living nearer or further from the centre.

Looking in detail at the results given in Table 1, it can be seen that increases in the costs of travel to shop, both the distance effect (g) and the fixed cost of shopping effect (w) decrease the total amount of travel, the average number of trips and the average distance travelled per period for the given group of shoppers. Again the average journey length cannot be determined from elementary reasoning and requires more detailed investigation. Increases in income (Y) and in inventory costs (c) work in the opposite direction to travel costs, and lead to increases in travel. Increases in density (D) have a rather different effect from increases in income — the total travel to the centre increases but the average number of trips per person, the average distance travelled and the average journey length are all unaffected. Finally an increase in the market area (T), with all other factors remaining constant, increases the total number of trips to the centre — more consumers are added, each of whom travels less than those already in the market area. As a result the average number of trips per individual in the market falls, while the average distance travelled and the average journey length rise.

This analysis shows that in the short run the effects of parameter shifts on travel characteristics can be largely predicted qualitatively without recourse to the evaluation of complicated integrals. Moreover, the results accord with an intuitive approach to the problem. Further consideration of these short-run effects reveals two other important features. Since the number of consumers remains the same, any increase in the total number of trips (N) is matched by an equiproportionate rise in the number of trips per person (AN) — the short-run elasticities of these two measures with respect to shifts in a

TABLE 2

Short-run Elasticities of Travel Characteristics

	N	*AN*	*AD*	*AJ*
a	0	0	0	0
b	0	0	0	0
c	0.490	0.490	0.485	–0.006
g	–0.280	–0.280	–0.353	–0.076
w	–0.284	–0.284	–0.213	0.073
Y	0.588	0.588	0.594	0.007
D	1	0	0	0

given parameter will be equal. Furthermore, since an increase in density does not change individual behaviour, the total number of trips is proportional to density and its short-run elasticity will be unity.

Finally, we can exploit a general relation between the various measures of travel behaviour. Since from (7) $AJ = AD/AN$, it follows that, for small changes in an exogenous variable, the elasticity of AJ with respect to this parameter is equal to the elasticity of AD less that of AN with respect to the same parameter.

In order to obtain a more precise assessment of the relative importance of the various determinants of the travel to shop, we need to evaluate the functions (12) to (16), using the integrals given in the Appendix. Since there are seven parameters in the full model, it is necessary to choose a base case and then vary one parameter at a time, holding all the others constant at the base level. The base values that we use are ($a = 4$, $b = 0.4$, $c = 0.1$, $g = 0.2$, $w = 0.2$, $Y = 5$, $D = 1$). In the short run the market boundary T is fixed, and we use the value which corresponds to the long-run equilibrium for the base case parameters ($T = 2.95$). Rather than give separate tables for variations in each parameter, we chose values equidistant on either side of the base case, calculated the associated travel characteristics, and from these calculated the arc elasticities. Experiments over a wider range of values suggested that these elasticities are fairly robust with respect to the values of the base case. The results are shown in Table 2.

This table, which is calculated for small changes rather than for differentials, does confirm what we have said earlier. The cost parameters have no short-run effect, and density affects only the total number of trips per period. The effect of all the other variables is substantial — income in particular has a large positive effect on the average frequency, but the travel costs and shopping costs both depress the average number of trips, while inventory costs have a strong impact on the increase in the number of trips. The one measure for which the *a priori* qualitative effects were unpredictable, the average journey

TABLE 3

The Effects of Parameter Shifts on Equilibrium Market Areas

T_y – (?),	T_a +,	T_b 0,	T_c +/–,	T_s +/–,	T_w +/–,	T_D –

length (*AJ*), is very insensitive to short-run parameter changes — this is because within the market area the effects of parameter shifts seem to be rather similar for all consumers, so that the average number of trips and the average distance travelled move very closely together.

Once we turn to long-run analysis, when the market area itself adjusts to the parameter shifts, T becomes an endogenous variable, so that we need to add together the effects of changes in individual behaviour and changes in the numbers of people served by a given centre. Furthermore, the parameters of the supply side (costs of running the shops) also have an impact on the market area, and hence on the average travel characteristics.

The long-run effect of changing some parameter (k) on a travel characteristic (H) is given by the total differential:

$$dH/dk = H_k + H_T T_k \qquad (20)$$

The signs of the partial derivatives H_k and H_T have already been given in Table 1, but we require also the signs of the effects of parameter shifts on equilibrium market area T_k. These results have been derived in Bacon (1989), and are merely reported here in Table 3. The income derivative was analysed by numerical methods and found to be negative over a large range of values, but for extreme parameter combinations there is a possibility that its sign is reversed. The effects of travel costs, shopping costs and inventory costs are negative except when they become large relative to income; then their effect on market area is positive.

Putting together the results of Tables 1 and 3, it can be seen that simple predictions of the long-run effects of parameter shifts on travel are for the most part qualitatively indeterminate. The effects of changes in the fixed and variable costs of supply are predictable, since they affect only market area and not the behaviour of individual shoppers. An increase in fixed costs increases market area and so leads to an increase in the number of trips to a typical centre (of which there will be fewer), raises the average journey length and the average distance travelled per unit time, and lowers the average number of trips made per shopper per period. Changes in marginal costs leave market areas unaltered, and therefore leave all travel characteristics unchanged. An increase in population density lowers the average number of trips per user, but its other effects are indeterminate. All other parameter shifts leave long-run travel behaviour qualitatively indeterminate — the forces acting on market area and on individual behaviour pull in opposite directions. In order to make quantitative assessments of these long-run effects we first solve, for a given set of parameters, the value of the market boundary in equilibrium,

TABLE 4.1

The Effects of Shifts in Fixed Cost of Supply

a	*T*	*N*	*AN*	*AD*	*AJ*
1.0	1.22	1.43	1.17	0.67	0.57
3.0	2.44	2.47	1.01	1.10	1.09
4.0	2.95	2.84	0.96	1.25	1.30
5.0	3.43	3.17	0.92	1.38	1.49
7.0	4.32	3.73	0.86	1.59	1.85
9.0	5.16	4.20	0.81	1.77	2.17

TABLE 4.2

The Effects of Shifts in Inventory Costs

c	*T*	*N*	*AN*	*AD*	*AJ*
0.05	3.48	2.33	0.67	1.02	1.52
0.10	2.95	2.84	0.96	1.25	1.30
0.15	2.70	3.20	1.18	1.41	1.19
0.25	2.45	3.72	1.52	1.65	1.08
0.35	2.31	4.11	1.78	1.82	1.03
0.45	2.23	4.44	1.99	1.97	0.99

TABLE 4.3

The Effects of Shifts in Travel Costs

g	*T*	*N*	*AN*	*AD*	*AJ*
0.10	3.76	4.05	1.08	1.83	1.70
0.20	2.95	2.94	0.96	1.25	1.30
0.30	2.59	2.31	0.89	0.99	1.12
0.50	2.23	1.77	0.79	0.74	0.93
0.70	2.03	1.48	0.73	0.61	0.83
0.90	1.90	1.29	0.68	0.52	0.77

TABLE 4.4

The Effects of Shifts in Shopping Costs

w	*T*	*N*	*AN*	*AD*	*AJ*
0.00	2.57	4.82	1.88	1.57	0.84
0.10	2.78	3.23	1.16	1.36	1.17
0.20	2.95	2.84	0.96	1.25	1.30
0.30	3.10	2.62	0.84	1.18	1.39
0.50	3.35	2.34	0.70	1.08	1.55
0.70	3.57	2.16	0.61	1.01	1.67
0.90	3.77	2.04	0.54	0.96	1.77

TABLE 4.5

The Effects of Shifts in Income

Y	*T*	*N*	*AN*	*AD*	*AJ*
2.5	4.15	2.45	0.59	1.05	1.77
5.0	2.95	2.84	0.96	1.25	1.30
7.5	2.49	3.12	1.25	1.39	1.11
12.5	2.04	3.52	1.73	1.59	0.92
17.5	1.81	3.82	2.11	1.74	0.83
22.5	1.66	4.07	2.45	1.86	0.76

TABLE 4.6

The Effects of Shifts in Density

D	*T*	*N*	*AN*	*AD*	*AJ*
1.0	2.95	2.84	0.96	1.25	1.30
2.0	1.88	4.05	1.08	0.92	0.85
4.0	1.22	5.73	1.17	0.67	0.56
6.0	0.96	7.02	1.22	0.55	0.45
8.0	0.81	8.10	1.25	0.48	0.38
10.0	0.71	9.05	1.27	0.43	0.34

TABLE 5

Long-run Elasticities of Travel Behaviour
($a = 4$, $b = 0.4$, $c = 0.1$, $g = 0.2$, $w = 0.2$, $Y = 5$, $D = 1$)

	T	N	AN	AD	AJ
a	0.675	0.496	–0.186	0.452	0.620
c	–0.252	0.315	0.551	0.321	–0.243
g	–0.369	–0.547	–0.192	–0.596	–0.411
w	0.109	–0.205	–0.320	–0.142	0.172
Y	–0.500	0.241	0.717	0.279	–0.458
D	–0.665	0.527	0.176	–0.456	–0.628

and then use the integrals from the Appendix to evaluate the travel characteristics for this set of parameters and market boundary. We use the same base set of parameters as for the short-run elasticities. In addition to calculating the arc elasticities, we also give tables of travel behaviour for a range of values for each parameter in turn — this allows us to assess how far the base values chosen are representative, as well as to see the response over a wider range of parameter shifts. The results are shown in Tables 4.1 to 4.6 (there is no table for the marginal cost of supply, since it has no long-run effect on travel behaviour).

The tables not only give information on the direction of the long-run parameter impacts on travel characteristics; they also allow us to see the relative quantitative importance of the different factors. In order to summarise the results in the tables and to put them on a comparable basis, we calculate elasticities around the basic values of the parameters which are held constant while each parameter is varied in turn. These results are shown in Table 5.

We comment on the main features of the results, analysing each of the aspects of travel behaviour, but we begin with the effects on market area itself.

The equilibrium market area (T)

The equilibrium market area is very sensitive to changes in certain parameters. In particular, a rise in fixed costs produces larger areas and fewer shops. An opposing tendency is that higher income or higher density leads to smaller market areas and more shops. The variable costs of travel to shop have an important effect in reducing market area, so that the average shopper in a smaller market area will tend to shop more frequently than the average shopper in a larger market area. This tendency will of course be countered in part or totally by the reduction in the frequency of shopping for each individual. Both the fixed cost of shopping and the inventory cost have rather less effect on market area, and increases in the fixed cost are associated with an increase in total market area.

The number of trips to the centre per period (*N*)

The total number of trips to the centre increases with the fixed costs of supply, inventory costs of holding shopping, income and density; it decreases with the fixed costs of going shopping and the costs of travel. The elasticities are all substantial, but it is interesting that the value for density is almost twice as great as that for income — as income rises the number of trips rises less fast, since the consumer can manage larger inventories as well. Again the fixed cost of shopping is less important than the variable cost, which is the strongest factor in altering the total number of trips to a centre. Comparing these results with the short-run elasticities given in Table 2, we see that qualitatively they have the same impact. However, inventory costs, shopping costs and income all have lower long-run elasticities, while travel costs are much more important in the long run — not only does each person shop less, but there are also fewer shoppers visiting the centre.

The average number of trips per period per shopper (*AN*)

The average number of shopping trips per period shows a wide range of elasticities. For the fixed costs of supply, travel costs and the fixed costs of shopping, the effect is negative, while for density, income and inventory costs it is positive. The first striking feature is the sign of the effect of an increase in the fixed costs of supply. The market area increases, and so does the number of trips to the centre, but the average per shopper falls — the extra shoppers are all further away and have lower frequencies. The relative weights of income and density are the opposite of their relative effects on the total number of shoppers — density is more important in adding shoppers, and therefore trips, but does not alter the characteristic of the average shopper so much. Increases in income raise the frequency of trips by each shopper, as well as reducing market areas; this leaves the highest frequency shoppers attached to the centre. The fixed costs of shopping are now more important than the variable travel costs, and this highlights their importance for certain aspects of the model. The long-term elasticities are generally larger than the short-term values, except for travel costs, but the differences between the long- and short-term responses for the average number of trips per shopper is relatively small.

The average distance travelled per period per shopper (*AD*)

The average distance travelled per period, which is the sum of all shopping journeys per period averaged over all consumers, increases with the fixed costs of supply, inventory costs, income and density. It decreases with the fixed and variable costs of shopping. As expected, the travel costs are particularly important — the decrease in market area removes the consumers who are travelling furthest, and for all remaining consumers the frequency of shopping is reduced. The effects of income and density work in opposite directions for this measure of travel behaviour. Increases in income reduce market area but increase frequency for those remaining: the latter effect is stronger, so that the average distance travelled rises. Density reduces market area and leaves individuals unchanged, so that the average distance must fall. Again the short- and long-run elasticities are fairly similar — travel costs are more important in the long run, while income is less important.

The average journey length per period per consumer (*AJ*)

The average length of the journey to shop weights not only the distance each shopper has to travel but also the number of times each does so — it is the ratio of the total distance travelled to the total number of trips, rather than the ratio of total distance to the number of consumers (AD). Inspection of the table of elasticities shows that these two measures behave quite differently for certain of the parameters. Increases in the fixed cost of supply are similar for both — the increase in market area, while individual behaviour is unchanged, raises both the average distance travelled and the average journey length. Increases in inventory costs and income shorten the average journey to shop, while increasing the average distance travelled to shop. The decreasing market area leaves shoppers who are nearer to the centre but shop more often; this has a strong depressing effect on the average journey, but not so strong on the average distance travelled. In addition, an increase in income or in inventory costs raises the frequency of shopping for those still in the market area. The relative importance of this latter effect depends on its strength at various distances — if all consumers had their frequency raised equally, then the average journey length would be unchanged but the average distance travelled would rise. Only if frequency rose more strongly for greater distances could the effect on average journey to shop be an increase. The effect of the increase in density is similar to that of the fixed costs of supply — the reduction in market area with unchanged individual behaviour reduces both the average distance travelled and the average journey length. The cost of travel continues to have a depressing effect on average journey length, but the fixed cost of shopping now has a small positive effect (rather than the small negative effect it has on average distance). For this measure of travel behaviour there is a major difference between the long- and short-run elasticities — all the short-run effects are negligible, while the long-run effects are substantial. The change in the size of the market affects the average journey length by adding (or subtracting) customers with longer (or shorter) average journeys, while the average for the existing customers is scarcely affected by the parameter shifts.

These results show that travel behaviour for the journey to shop can be related to various economic factors through a utility maximising model. In comparing these theoretical results with actual experience, a number of points must be borne in mind. Some of the variables (for example, inventory costs and density) change only slowly over time, so it is only in the really long run that their influence can be detected. The introduction of the refrigerator and the freezer probably had an important effect on the frequency of shopping by reducing inventory costs caused by losses on perishability. But in developed countries these gains have long been realised, and *variations* in inventory costs are not likely to be important at present. Similarly, changes in availability of transport will in aggregate be slow moving, so they may explain relatively little in a time series context if the time period is not very lengthy. In a cross-section household survey the ownership of a car might indeed be an important factor in determining the relative frequency of shopping. The principal factor which is likely to vary by a substantial amount is income, and both the long- and short-run elasticities in the theoretical model were found to be

substantial. This is partly because income as defined is all spent on shopping for the single composite good (either directly or indirectly). In a more general model with other claims on income (for example, rent and non-shopping expenditure) the elasticity of travel with respect to total income could be considerably lower.

An important finding of the paper is that the sensitivity of the total amount of travel to shop is quite different in the short run and the long run. As changes are introduced which alter consumer behaviour, the equilibrium size of the market changes, and this increases the elasticities with respect to certain variables. However, the process of adjustment is slow — shops leave or enter the market or relocate only as fixed costs become variable and new sites become available. We also showed that the different measures of travel behaviour had different elasticities — in particular, the length of the average journey to shop is very insensitive to parameter changes in the short run, while the average number of shopping trips and the average distance travelled per period both have substantial and similar short-run elasticities.

The effect of transport costs on the journey to shop is a key factor for transport planning, since this is the variable which can most easily be influenced by policy changes. In the short run the elasticities with respect to the elements of distance-related (variable) cost and trip-related (shopping) cost are almost identical, but in the long run they are quite different. Indeed, for certain travel characteristics (the total number of trips and the average journey length) they are even of opposite signs. Thus to obtain the correct policy prescriptions we need to specify carefully not only the horizon for planning but also the nature of the structure of travel costs, as well as the appropriate dimension of travel.

CONCLUSIONS

In this paper we have taken a utility maximising model of consumer shopping behaviour in which the frequency of shopping is endogenous. By aggregating the demands over all consumers we generated expressions for the total number of shoppers using a shopping centre and the frequencies at which they do so. For a given set of shopping centres we then derived the total number of trips per period to the centre and the average number of trips per consumer, the average distance travelled to shop per period and the average journey length per shopper. These travel characteristics are functions of the parameters of the individual demand functions (income, inventory costs, travel costs) and the exogenous density of households. Elasticities of travel behaviour, for the given market size, with respect to these parameters were then calculated. The elasticities for average distance and the average number of trips were very similar, but those for average journey length were all near zero.

The second part of the paper looked at the long run, in which shops relocate in order to maximise profits as economic circumstances change (and shops enter or exit until a zero profit equilibrium is reached). The effects of shifts in the size of market area was then incorporated into the analysis of the effects of changes in parameter values on the travel

to shop. The long-run elasticities were found to be very different from the short-run values — often being of the opposite sign. Moreover, the various measures of average travel characteristics yielded quite different long-run elasticities, even when the short-run values were very similar. These results emphasised the need to specify the time horizon and the measure of travel behaviour under consideration when making policy changes designed to affect the journey to shop.

APPENDIX

The Derivation of Aggregate Travel Characteristics

The frequency of shopping for an individual living t units from the shopping centre is:

$$f(t) = -c + c\{1 + y/[c(gt + w)]\}^{0.5} \quad \text{(A1)}$$

and we require to evaluate:

$$\int_0^T f(t)\,dt \quad \text{and} \quad \int_0^T t\,f(t)\,dt \quad \text{(A2)}$$

Clearly the difficulty is to find the integral of the second term in (A1). The substitution

$$x^2 = c(gt + w)$$

is used so that we require between the appropriate limits:

$$\int (x^2 + Y)^{0.5}\,dx$$

Using the standard formulae (Beyer, 1987) we obtain for the first integral:

$$-cT + \{[c(gT + w)]^{0.5}\,[c(gT + w) + Y]^{0.5} - (cw)^{0.5}\,(cw + Y)^{0.5} + Y \log (\{[c(gT + w)]^{0.5} + [c(gT + w) + Y]^{0.5}\}/[(cw)^{0.5} + (cw + Y)^{0.5}])\}$$

For the second integral the same substitution yields:

$$\int (2/cg^2)(x^2 - cw)(x^2 + Y)^{0.5}\,dx$$

The term multiplying by $(-cw)$ uses the same formula as above whilte the integral

$$\int x^2(x^2 + Y)^{0.5}$$

is evaluated using Beyer. Hence the second of the integrals becomes:

$$-cT^2/2 - (w/g^2)\{[c(gT + w)]^{0.5}[c(gT + w) + Y]^{0.5} - (cw)^{0.5}(cw + y)^{0.5} + Y \log (\{[c(gT + w)]^{0.5} + [c(gT + w) + Y]^{0.5}\}/[(cw)^{0.5} + (cw + Y)^{0.5}])\}$$
$$+ (2/cg^2)\{[c(gT + w)]^{0.5}[c(gT + w) + Y]^{1.5}/4 - (cw)^{0.5}(cw + Y)^{1.5}/4$$
$$- (Y/8)[c(gT + w)]^{0.5}[c(gT + w) + Y]^{0.5} + (Y/8)(cw)^{0.5}(cw + Y)^{0.5}$$
$$- (Y^2/8) \log (\{[c(gT + w)]^{0.5} + [c(gT + w) + Y]^{0.5}\}/[(cw)^{0.5} + (cw + Y)^{0.5}])\}$$

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