BENEFIT-COST RULES FOR URBAN TRANSIT SUBSIDIES

An Integration of Allocational, Distributional and Public Finance Issues

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1. INTRODUCTION

This paper considers rules for assessing the benefits and costs of urban transit subsidies, taking account both of the allocational and of the income-distributional implications of public transport services. Within the framework of standard benefit-cost analysis, subsidies provide consumer surplus benefits to riders, and external benefits in the form of reduced highway congestion. However, the benefits may accrue to different income groups, and decision-makers may therefore wish to apply different weights to them. There will also be allocational costs of financing the subsidies through alternative forms of taxation; and the method of funding will itself have distributional implications which must be taken into account if a complete picture of the effect of the subsidies is to be achieved. Further, the administration determining fare and service levels may itself be funded by another (central, Federal, or regional) level of government, and may view the cost of funds raised locally differently from that of central funds.

If distributional and financing issues were ignored, the economically efficient, first-best, course for urban transport would be to set prices of both public and private transport equal to their respective marginal social costs.[1] With congested conditions on urban roads, this would require automobile users to pay congestion charges to cover the external costs of congestion imposed at the optimal flow of traffic. If administrative, political or other factors prevented the introduction of a road pricing scheme, the second best course would be to set prices on the substitute mode, public transport, below public transport marginal costs. Glaister and Lewis (1978) estimated optimal second-best peak and off-peak

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[1] Marginal cost pricing of transit services would have to allow for the marginal delay costs imposed on existing riders. See Turvey and Mohring (1975).

public transport fares on different modes in London within this type of second-best efficiency framework.

Subsequently Glaister (1984) developed a model to evaluate the efficiency costs and benefits of changes in subsidy levels, in circumstances where subsidies could be used to finance either reductions in fares or increases in level of service. This model has been used by the United Kingdom Department of Transport (1982) and by the English Metropolitan Counties in the evaluation of different levels of financial support for public transport services. However, the model does not take account of distributional factors in the measurement of the benefits of public transport services; nor does it consider the efficiency and distributional impact of the increased taxation required to fund subsidised transit services, or the question how far those who decide the level of subsidy are influenced by the source of the subsidy funds.

In this paper we derive benefit-cost rules for fare reductions and increases in services which do incorporate distributional issues and which do allow for the opportunity cost, or shadow price, of the funds required to fund the increased subsidies.[2] Our analysis of distributional considerations follows earlier work by Feldstein (1972), but also allows for the impact of the financing of subsidies on distribution, thus handling equity considerations within a whole budget framework.[3]

Since decisions on fare and service levels in the UK (and in many other countries) are taken by local jurisdictions, we concentrate on benefit-cost rules which would lead to maximisation of local, rather than national, social welfare. This enables us to highlight the way in which the system of local government finance will cause a divergence of local and national interests on optimal fare, service and subsidy levels.

Part 2 of the paper considers benefit-cost rules evaluating transit fare reductions in the absence of external benefits from transit services. Part 3 extends these rules to incorporate the benefits of reduced highway congestion. Part 4 develops rules to assess the benefits of using transit subsidies to finance improved service frequency. Part 5 summarises our main conclusions.

2. THE COSTS AND BENEFITS OF FARE REDUCTIONS

The budget constraint of the local jurisdiction

Subject to approved changes in balances, local jurisdictions in the United Kingdom are required to balance their current budgets without borrowing. Local finance is derived from property taxation ("the rates"), grants in aid from central government, and revenues from traded services. The local tax base X_0 is the average value of assessed property rentals, where property is defined as land and fixed structures. Its after-tax price is $q_0 = p_0 + t_0$, where p_0 is a unit producer

[2] For discussion of the concept of the shadow price of public funds see Ballard, Shoven and Whalley (1985) and Stuart (1984).

[3] Boadway (1976) also indicates how Feldstein's distributional characteristic can be incorporated into project appraisal.

price and t_0 is the amount of tax on that unit. Total tax revenue is therefore $T = t_0 X_0$. The grant situation is complicated, but, up to some specified limits which we assume are not breached, deficits on transit operations count as relevant expenditure for grant aid. Some fraction g of this deficit is financed from central funds, which the locality regards as free money.

A welfare-maximising local government determines and provides a fixed quantity of public services for which no charge is made. We denote the cost of this, net of any block grant that the local authority receives from central government, as z. In addition, the local government markets s locally traded services. Consequently its budget constraint is

$$t_0 X_0 = (1 - g)\left\{ \sum_{k=1}^{s} [F_k + C(X_k) - q_k X_k] + z \right\}, \qquad (0)$$

where F_k is the fixed cost of facility k and $C(X_k)$ is its total variable cost. We assume that non-traded services dominate the budget.

Social welfare in the local area

A representative household has a utility function $u(x, z)$, where x is an $n + 1$ ($n > s$) vector of traded goods and services. The consumer's budget constraint is $m = q'x$, where m is lump-sum income, assumed to be unaffected by local activities, and q is a vector of $n + 1$ prices confronted in market places. The demand functions $x(q, z, m)$ derived from individual maximisation can be substituted back into the utility function to yield the indirect utility function $\psi(q, z, m)$.

Suppose households are homogeneous in tastes but differ in income. Let the discrete distribution of incomes be approximated by a continuous distribution, which we will specify later. For any value of m, the probability that the value of a randomly selected income is near m is

$$\int_{m-\Delta m}^{m+\Delta m} f(m)\, \mathrm{d}m$$

and

$$E(m) = \int_0^\infty m f(m)\, \mathrm{d}m$$

is the expected value of m.

Thus the utility accruing on average to households in the jurisdiction is

$$\bar{\psi} = \int_0^\infty \psi(q, z, m)\ f(m)\, \mathrm{d}m.$$

Total welfare W can then be calculated after each household's level of utility has been multiplied by $\beta'(m)$, which is the jurisdiction's evaluation of a marginal unit of utility accruing to that particular household. This evaluation of marginal utility is usually a decreasing function of income.[4] Hence

[4] Where the jurisdiction takes a distributionally neutral stance, in the sense of being indifferent to equal *utility* gains to different individuals, $\beta'(m) = 1$ at all levels of income.

$$W = N\int_0^\infty \psi\,(q, z, m)\,\beta'(m)\,f(m)\,\mathrm{d}m, \tag{1}$$

where N is the number of households in the community.

The impact of fare changes on welfare

A subsidy is justified if the change in total welfare, $\mathrm{d}W$, is positive. In view of the local government budget constraint, any change in the transit fare q_j must be accompanied by a change in the post-tax price of the locally-taxed good q_0. Thus

$$\mathrm{d}W = N\int_0^\infty \left(\frac{\partial\psi}{\partial q_j}\,\mathrm{d}q_j + \frac{\partial\psi}{\partial q_0}\,\mathrm{d}q_0\right)\beta'(m)\,f(m)\,\mathrm{d}m\,. \tag{2}$$

From Roy's theorem, $x_j = -(\partial\psi/\partial q_j/\partial\psi/\partial m)$, and so $\mathrm{d}W > 0$ if

$$-\mathrm{d}q_j\int_0^\infty x_j\,(q, z, m)\,\alpha'(m)\,\beta'(m)\,f(m)\,\mathrm{d}m > \mathrm{d}q_0\int_0^\infty x_0(q, z, m)\,\alpha'(m)\,\beta'(m)\,f(m)\,\mathrm{d}m, \tag{3}$$

where $\alpha'\,(m)$ represents the private marginal utility of income, $\partial\psi/\partial m$. Benefits of the change are calculated on the left hand side of (3) and costs on the right hand side.

Following Feldstein (1972), we denote the "distributional characteristic" of good j as ρ_j, where

$$\rho_j = \frac{N}{X_j}\int_0^\infty x_j\,(q, z, m)\;\mu'(m)\,f(m)\,\mathrm{d}m, \tag{4}$$

where X_j denotes aggregate demand for good j and where $\mu'\,(m) = \alpha'\,(m)\,\beta'\,(m)$ is the socal marginal utility of income; this again is a function (usually decreasing) of m. Substituting (4), together with the similar expression for X_0, into (3) gives an increase in total welfare, W, if

$$-\rho_j\,X_j\,\mathrm{d}q_j > \rho_0\,X_0\,\mathrm{d}q_0.$$

If we denote ρ_j/ρ_0 by Δ, the benefit-cost criterion indicates that fares should be reduced if

$$-\Delta\frac{X_j\,\mathrm{d}q_j}{X_0\,\mathrm{d}q_0} > 1\,. \tag{5}$$

The negative sign makes the expression positive, because $\mathrm{d}q_j < 0$. In (5) Δ is the ratio of the distributional characteristic of the subsidised service to that of the taxed good. Because in (4) ρ_j is greater for a necessity than for a luxury, Δ in (5) is greater than one if the income elasticity of demand for the taxed good exceeds that for the subsidised service. The value of Δ, as will be made clear below, is affected by the parameters of the income distribution.

Next equation (0) is totally differentiated. Since the local government is constrained to balance its budget, q_0 will have to change if q_j is reduced. The other

$s - 1$ locally determined prices are held constant. To focus on issues of practical importance, strong separability is invoked between, but not within, sectors

$$t_0 \frac{\partial X_0}{\partial t_0} \, dt_0 + X_0 dt_0 = (1-g)\left\{ \sum_{k=1}^{s} \left[C'(X_k) \frac{\partial X_k}{\partial q_j} \, dq_j - q_k \frac{\partial X_k}{\partial q_j} \, dq_j \right] - X_j dq_j \right\}.$$

Noting $dq_0 = dt_0$ and letting $\tau_0 = t_0/q_0$:

$$X_0 dq_0 \left[1 + \tau_0 \frac{\partial X_0}{\partial q_0} \frac{q_0}{X_0} \right] = -X_j dq_j \, (1-g) \left[1 + \sum_{k=1}^{s} \frac{q_k - MC_k}{q_k} \frac{\partial X_k}{\partial q_j} \frac{q_j}{X_k} \frac{q_k X_k}{q_j X_j} \right].$$

Price elasticities are denoted by η, the divergencies from marginal costs on the right hand side (which may be subsidies or "tax rates") are expressed as τ, and total sales revenue is $R_k = q_k X_k$:

$$-X_0 dq_0 \, [1 + \tau_0 \eta_{00}] = X_j dq_j \, (1-g) \, [1 + \tau_j \eta_{jj} + \sum_{k=2}^{s} \tau_k \eta_{kj} \, (R_k/R_j)]. \qquad (6)$$

In (6), the expression for good j on the right hand side has been isolated because it is the focus of attention. The expression on the far right is the change in surpluses and deficits of other trading activities within the locality's jurisdiction. When we rearrange (6) and substitute into (5), the benefit-cost criterion now indicates that transit fares should be reduced if

$$\frac{\Delta}{1-g} \frac{1 + \tau_0 \eta_{00}}{1 + \tau_j \eta_{jj} + \sum_{k=2}^{s} \tau_k \eta_{kj} (R_k/R_j)} > 1. \qquad (7)$$

A diagrammatic exposition of this criterion (though one which ignores distributional considerations and the spillover effects on other local government goods and services) is provided in Dodgson and Topham (1986).

Making distributional issues operational

For equation (7) to be of practical use, the distributional parameter Δ has to be made operational. To do this it is necessary to specify functional relationships for the constituents of equation (4). Following Feldstein (1972), household demands are represented by iso-elastic functions, $x_j = h_j m^{\theta_j}$, and the social marginal utility of income is specified to be a simple Engel function, $\mu'(m) = m^{-\delta}$.

Substituting these relationships into (4):

$$\rho_j = \frac{\int_0^\infty m^{(\theta_j - \delta)} f(m) dm}{\int_0^\infty m^{\theta_j} f(m) dm}. \qquad (8)$$

We take the relative density function $f(m)$ to be normally distributed. The log of income is approximately normally distributed, and in such circumstances Feldstein (1972, p. 35) showed that $\rho_j = \exp[-\delta\omega + \frac{1}{2}(\delta^2 - 2\theta_j\delta)\sigma^2]$, where ω is the mean of the log m and σ^2 is the variance about that mean. Thus

$$\Delta = \rho_j/\rho_0 = \exp[\delta\sigma^2(\theta_0 - \theta_j)]. \quad (9)$$

In (9) σ^2, the variance of $\log m$, is observed only indirectly. Noting that for the lognormal distribution the mean, $\bar{m}$, equals $\exp(w + \frac{1}{2}\sigma^2)$ and the variance, $v(m)$, equals $\exp(2w + \sigma^2)[\exp(\sigma^2)-1]$, we have $1 + [v(m)/\bar{m}^2] = \exp(\sigma^2)$.[5] Therefore

$$\Delta = \rho_j/\rho_0 = \left\{1 + \left[v(m)/\bar{m}^2\right]\right\}^{(\theta_0 - \theta_j)\delta}. \quad (10)$$

Inserting (10) into (7) the benefit-cost criterion now shows that fares should be reduced if

$$\frac{\left\{1 + [v(m)/\bar{m}^2]\right\}^{(\theta_0 - \theta_j)\delta}}{1 + \tau_j\eta_{jj} + \sum_{k=2}^{s} \tau_k\eta_{kj}(R_k/R_j)} > \lambda(1-g), \quad (11)$$

where $\lambda = 1/(1 + \tau_0\eta_{00})$, and $\lambda(1-g)$ is the local marginal efficiency cost of public funds.

Evaluation of (11) requires evidence on own-price elasticities (η_{00}, η_{jj}), cross-price elasticities between transit and other locally-traded services (the η_{kj}'s), income elasticities (θ_0, θ_j), tax or subsidy rates (τ_0, τ_j, and the τ_k's), income mean and variance $[v(m)/\bar{m}^2]$, the proportion of local expenditure funded by central government (g), the elasticity of the social marginal utility of income (δ) and the relative revenue from transit and other locally traded services (the R_k/R_j's).

It is unlikely that direct evidence will be available on cross-elasticities between transit and other locally traded services; but the cross-elasticities are likely to be small, and, since substitution is a more common relationship than complementarity, in most cases positive. In the UK most locally traded services do not cover their costs, so the τ_k terms are likely to be negative. This means that the interdependence term

$$\sum_{k=2}^{s} \tau_k\eta_{kj}(R_k/R_j)$$

will be negative, but small. This will serve to increase the justification for fare reductions by reducing the size of the denominator on the left hand side of (11). The explanation of this second-best impact is that the subsidisation of locally traded services creates a distortion, so that a reduction in the demand for those services as a result of a reduction in transit fares will reduce the impact of the distortion.

We can be more definite on the size of the other (more important) terms in (11). Evidence suggests a value for the own-price elasticity of demand for owner-occupied housing (η_{00}) of around -0.60, and for transit services (η_{jj}) of around -0.30. The income elasticity of demand for housing (θ_0) may be around $+0.80$ and for transit (θ_j) zero or even negative. The rate of local tax on owner-occupied housing (τ_0) is around 0.20. For τ_j we require the marginal subsidy rate

[5] See Johnson and Leone (1977 pp. 142–143) for discussion of the properties of the log-normal distribution.

for additional passengers on the transit system. For illustrative purposes we simply assume that marginal social costs per passenger are equal to average system-wide costs per passenger. In practice, of course, marginal costs may well differ from average costs, particularly if there is spare capacity on the transit system. In recent years the average subsidy (negative tax) rate on transit in the largest provincial English cities has varied between about -2.00 and about -0.67. The national value for $v(m)/\bar{m}$ [2] in 1982 was 0.37; a typical value for the proportion of local expenditure funded by central government (g) is 0.50; and Stern (1977, p. 243) has noted that for the elasticity of the social marginal utility (δ) a value of 2.00 has emerged from a number of UK studies.

These illustrative figures yield a value for the marginal cost (shadow price) of public funds at the national level (λ) of 1.14, and for the marginal cost of public funds to the local jurisdiction (the right hand side of (11)) of 0.57. If we ignore the interdependence term between transit and other locally traded services, the range of values quoted above for the transit subsidy rate (τ_j) yields a range of values for the left hand side of (11) of between 1.03 and 1.38. As we noted, the effect of the interdependence term is likely to be to increase these values slightly. Thus the benefit-cost criterion is, on these illustrative values, satisfied and a fare reduction would be justified.

3. THE BENEFITS OF REDUCED TRAFFIC CONGESTION

Lower transit fares also provide external benefits in the form of reduced congestion costs of private vehicles, because some car users are diverted to transit. These external benefits can be modelled as a fall in the "price",[6] q_a, of good a (automobile travel) for all car users who continue to demand it.

For our representative household the change in indirect utility is equal to

$$\left(\frac{\partial \psi}{\partial q_a}\ \frac{\partial q_a}{\partial X_a}\ \frac{\partial X_a}{\partial q_j}\right) \mathrm{d}q_j\,, \tag{12}$$

where $\partial X_a/\partial q_j$ shows the impact of changes in public transport fares on the total demand for car trips, and $\partial q_a/\partial X_a$ shows the impact of changes in traffic flows on the "price" of car trips. If we add this term (showing the benefit from reduced congestion) to the change in welfare in (2) and use Roy's theorem to derive individual demand for car trips, x_a, equation (3) can be modified to incorporate congestion benefits, so that $\mathrm{d}W$ is positive if

$$\begin{aligned} -\mathrm{d}q_j \int_0^\infty \left[x_j(q, z, m) + x_a(q, z, m)\ \frac{\partial q_a}{\partial X_a}\ \frac{\partial X_a}{\partial q_j}\right] \mu'(m) f(m) \mathrm{d}m \\ > \mathrm{d}q_0 \int_0^\infty x_0(q, z, m) \mu'(m) f(m) \mathrm{d}m. \end{aligned} \tag{13}$$

Incorporating a term ρ_a similar to the term for good j in (4) to represent the

[6] This "price" is a generalised cost covering both money and travel time. Issues involved in the derivation of values of travel time within a utility-maximising framework are discussed in section 4 of the paper.

distributional characteristics of car use, (13) can be rearranged to yield dW as positive if

$$-\rho_j X_j \, dq_j - \rho_a X_a \, dq_j \frac{\partial q_a}{\partial X_a} \frac{\partial X_a}{\partial q_j} > \rho_0 X_0 dq_0 .$$

The benefit-cost criterion is then to reduce fares if

$$-\Delta \frac{X_j \, dq_j}{X_0 \, dq_0} - \Delta_a \frac{\left(X_a \, dq_j \frac{\partial q_a}{\partial X_a} \frac{\partial X_a}{\partial q_j} \right)}{X_0 dq_0} > 1 , \tag{14}$$

where $\Delta_a = \rho_a / \rho_0$ is the ratio of the distributional characteristic of the external benefit to the distributional characteristic of the taxed good. Equation (14) is therefore the version of the original benefit-cost criterion (5) which incorporates the benefits of reduced highway congestion.

Since the local government budget constraint in (0) and (6) is unchanged, (6) can be substituted in (14) to yield a revised benefit-cost criterion of

$$\frac{\Delta + \Delta_a \phi}{1-g} \quad \frac{1 + \tau_{00} \eta_{00}}{1 + \tau_j \eta_{jj} + \sum_{k=2}^{s} \tau_k \eta_{kj} (R_k / R_j)} > 1, \tag{15}$$

where ϕ, the congestion benefit term, equals

$$\frac{X_a}{X_j} \frac{\partial q_a}{\partial X_a} \frac{\partial X_a}{\partial q_j} .$$

Equation (15) is the revised version of equation (7).

A value for Δ_a can be derived by the same method as we used for Δ. Hence from (10)

$$\Delta_a = \rho_a / \rho_0 = \left\{ 1 + [v(m)/\bar{m}^2] \right\}^{(\theta_0 - \theta_a)\delta}, \tag{16}$$

where θ_a is the income elasticity of demand for private car travel.

We can now consider likely ranges of values for Δ_a and for ϕ. The value of θ_a is likely to exceed 1.0, since the demand for private automobile travel is known to increase proportionately faster than income. Since we previously used a value of 0.8 for the income elasticity of demand for owner-occupied housing in the UK, this means that $(\theta_0 - \theta_a) < 0$ and hence $\Delta_a < 1$.

The term ϕ is likely to be small. It depends upon the relative modal shares of car and public transport (X_a/X_j), the responsiveness of generalised car costs to changes in traffic flows $(\partial q_a / \partial X_a)$, and the responsiveness of car trips to changes in public transport fares $(\partial X_a / \partial q_j)$. The second term can be derived from speed-flow relationships and from operating cost formulae, while the last term is obviously closely related to the cross-elasticity of demand between public transport fares and private vehicle use,

$$\eta_{aj} = \frac{\partial X_a}{\partial q_j} \frac{q_j}{X_a} .$$

Information on traffic levels, speed-flow relationships for different types of

roads, operating cost formulae, values of time, public transport fare levels, and cross-elasticities for different cities were collated for the Department of Transport (1982) study of policies for public transport subsidy, for which the Glaister model was used. From these published data it was possible to derive estimates for ϕ of 0.003 for Greater Manchester and Merseyside, 0.008 for South Yorkshire, 0.010 for the West Midlands and 0.002 for West Yorkshire. There are a number of acknowledged difficulties with the Department of Transport data; but it does still follow, especially in view of the distributional consequences of benefits to private car users (i.e. $\Delta_a < 1$), that the addition of the congestion benefit term to the benefit-cost criterion has only a small effect on the overall justification for subsidies, at least in England's provincial conurbations.[7]

4. THE COSTS AND BENEFITS OF SERVICE IMPROVEMENTS

Time savings for transit users

Section 2 considered the use of subsidies to reduce fares. Subsidies may be used instead to improve the quality of service. This section considers the measurement of the costs and benefits of service improvements. We consider those service improvements which reduce waiting times by making the transit service more frequent.

The consumer chooses a mode of transport according to its price (q) and the time spent (w) in making journeys. In practice different types of time (in-vehicle time, waiting time, walking time) will be valued differently; but, since this paper deals only with waiting time, it is convenient to assume that other types of time are held constant. Assuming that the demand for goods other than x_j depends only on price and income, the demand function for x_j is

$$x_j = f(q_0, q_1, \ldots\ldots, q_j, w_j, \ldots\ldots, q_n, m).$$

Substituting the $n + 1$ demand functions into the consumer's utility function, we get the indirect utility function

$$\psi = \psi(q, w_j, z, m). \tag{17}$$

Hence we require an expression for the value of time $\kappa(m)$. Holding all prices constant, we ask what compensation the consumer would be required to make if, with utility unchanged, the service was improved and hence waiting time w_j reduced. From (17)

$$\frac{\partial \psi}{\partial w_j}\, \mathrm{d}w_j + \frac{\partial \psi}{\partial m}\, \mathrm{d}m = 0.$$

Therefore

$$\frac{\mathrm{d}m}{\mathrm{d}w_j} = -\frac{\partial \psi / \partial w_j}{\partial \psi / \partial m} = \kappa(m). \tag{18}$$

Note that $\partial\psi/\partial w_j$ is negative, since increases in time lower utility. Hence $\mathrm{d}m/\mathrm{d}w_j$ is positive and shows what the consumer is prepared to pay for a one-unit reduc-

[7] Traffic congestion is much more severe in London than in other British cities.

tion in time, or what he would be prepared to accept in compensation for a one-unit increase in time; hence equation (18), the marginal rate of substitution between time and income, shows the value of a unit of time. Since $\partial\psi/\partial m$ declines as income increases, this value is an increasing function of income.

The benefit-cost ratio for service improvements

A service is said to be improved when w_j is reduced. Like a reduction in fares, a fall in w_j requires increased government expenditure, which may have to be financed from local taxation. To justify the change we require an improvement in total welfare, W. From (1), incorporating (17), we hold all prices constant except that on the locally taxed commodity, and enquire whether

$$dW = N\int_0^\infty \left(\frac{\partial\psi}{\partial w_j}\, dw_j + \frac{\partial\psi}{\partial q_0}\, dq_0\right)\beta'(m)\, f(m)\, dm > 0. \tag{19}$$

This is so if

$$dq_0 \int_0^\infty \frac{\partial\psi}{\partial q_0}\, \beta'(m)\, f(m)\, dm > -dw_j \int_0^\infty \frac{\partial\psi}{\partial w_j}\, \beta'(m)\, f(m)\, dm. \tag{20}$$

If we use Roy's theorem for the left-hand side, multiply the right-hand side by $(\partial\psi/\partial m)/(\partial\psi/\partial m)$, and substitute in (18), dW is positive if

$$-dw_j \int_0^\infty \kappa(m)\mu'(m)\, f(m)\, dm > dq_0 \int_0^\infty x_0(q, z, m)\mu'(m)\, f(m)\, dm. \tag{21}$$

Equation (21) can be simplifed by using (4) for the right-hand side and the similar expression for the value of time to yield

$$-\rho_\kappa\, K\, dw_j > \rho_0\, X_0\, dq_0. \tag{22}$$

The term K is the aggregate monetary value of time savings. Letting $\Delta_\kappa = \rho_\kappa/\rho_0$, we have the benefit-cost criterion to increase service levels if

$$-\Delta_\kappa \frac{K\, dw_j}{X_0\, dq_0} > 1, \tag{23}$$

which is similar to (5). If $\kappa(m)$ is specified as a simple Engel function, from (10) it follows that

$$\Delta_\kappa = \rho_\kappa/\rho_0 = \left\{1 + [v(m)/\overline{m}^2]\right\}^{(\theta_0 - \theta_\kappa)\delta}, \tag{24}$$

where θ_κ is the income elasticity of the value of time.

Next (0) is differentiated again. On this occasion all prices are held constant except q_0. Following a similar procedure to that which led to (6), and assuming $\partial X_k/\partial w_j = 0$, we have

$$X_0\, dq_0\, (1 + \tau_0\eta_{00}) = -(1-g)(q_j - MC_j')\, dX_j, \tag{25}$$

where MC_j' is the additional cost of carrying an extra passenger generated by a service improvement; we take it to be

$$MC_j' = \frac{\partial C}{\partial B}\frac{\partial B}{\partial X_j},$$

where $\partial C/\partial B$ equals marginal cost per bus mile (B) and $\partial X_j/\partial B$ shows the impact of increases in bus-miles on demand. (We assume that increasing bus-miles provides the capacity to carry the additional demand generated, at no extra cost other than that of the increased bus-miles.)

From (25)

$$X_0\,\mathrm{d}q_0 = -\lambda(1-g)\left[q_j - \frac{\partial C}{\partial B}\,\frac{\partial B}{\partial X}\right]\mathrm{d}X_j\,, \tag{26}$$

where $\lambda = 1/(1 + \tau_0\eta_{00})$. Now when (26) is substituted into (23), we have on re-arrangement the benefit-cost criterion to increase service levels if

$$-\Delta_\kappa \frac{K\mathrm{d}w_j}{\left[\dfrac{\partial C}{\partial B}\,\dfrac{\partial B}{\partial X_j} - q_j\right]\mathrm{d}X_j} > \lambda(1-g), \tag{27}$$

where $\lambda(1-g)$ is the marginal cost of public funds to the locality. On the left-hand side of (27) note that, evaluating at the mean, $K = N\kappa(m)$ and $\mathrm{d}X_j = N\mathrm{d}x_j$.

Modelling the effects of increased bus-miles on waiting time

To reduce waiting times the operator increases bus-miles, B. This reduces the headway (H) between buses on a fixed route network. The time saving for a traveller depends on the number of journeys he takes multiplied by the reduction in waiting time as bus-miles are increased. Hence

$$\mathrm{d}w_j = x_j\,\frac{\partial w_j}{\partial B}\,\mathrm{d}B = x_j\,\frac{\partial w_j}{\partial H}\,\frac{\partial H}{\partial B}\,\mathrm{d}B\,, \tag{28}$$

where $\partial w_j/\partial H$ shows the impact of changing headway on waiting time, and $\partial H/\partial B$ shows the impact of changes in bus-miles on headway. On a network with a fixed mileage of route, M, average frequency is equal to the average number of buses passing a fixed point on the network in a particular direction during a given period of time. Therefore hourly frequency is equal to $B/(2Mh)$ where h is the number of hours for which the system operates and to which the bus-miles figure, B, relates. Average headway is simply the inverse of frequency, and so headway (measured in seconds) equals

$$H = \frac{3600(2Mh)}{B},$$

hence

$$\frac{\partial H}{\partial B} = -\frac{3600(2Mh)}{B^2} = -\frac{H}{B}\,. \tag{29}$$

The term $\partial w_j/\partial B$ can be derived from empirical studies relating average passenger waiting times to headway. For example, Seddon and Day (1974) derived the following waiting time relationship for bus passengers in Manchester:

$$w_j = 11.39 + 0.49H - 0.00009H^2\,, \tag{30}$$

where both w_j and H are measured in seconds. Hence from (29) and (30)

$$\partial w_j/\partial B = -(0.49H - 0.00018H^2)/B. \tag{31}$$

The change in journeys as a result of the service improvement is

$$dx_j = \frac{\partial x_j}{\partial B}\, dB. \tag{32}$$

Substituting (28), (31) and (32) into (27) gives the benefit-cost criterion that services should be improved if

$$\Delta_\kappa \frac{\kappa(m)(0.49H - 0.00018H^2)}{\frac{\partial C}{\partial B}\frac{B}{X_j} - q_j \epsilon} > \lambda(1-g), \tag{33}$$

where $\epsilon = (\partial x_j/\partial B)/(B/x_j)$ is the bus-mile service elasticity, the elasticity of trips with respect to changes in bus-miles, and X_j/B equals passenger journeys per bus-mile, a measure of service utilisation. As noted previously, $\kappa(m)$ equals the value of waiting time, H equals bus headway, $\partial C/\partial B$ equals marginal cost per bus-mile, and q_j equals fare per journey.[8]

The choice between fare reductions and service improvements

The optimal trade-off between fare reductions and service improvements, which is obtained from (7) and (33), is

$$\frac{1}{1 + \tau_j\eta_{jj} + \sum_{k=2}^{s} \tau_k\eta_{kj}(R_k/R_j)} = (\Delta_\kappa/\Delta) \frac{\kappa(m)(0.49H - 0.00018H^2)}{\frac{\partial C}{\partial B}\frac{B}{X_j} - q_j \epsilon}. \tag{34}$$

Equation (34) highlights the distributional implications of choosing between fare reductions and service improvements. From (10) and (24) the term Δ_κ/Δ equals $(\theta_j - \theta_\kappa)\delta$, where θ_κ is the income elasticity of the value of time. The most commonly derived empirical values of travel time are those estimated from studies of commuters' mode choice. On the basis of such studies, the UK Department of Transport adopted a standard value for evaluating waiting time equal to a constant proportion (0.5) of income. Time values which are a constant proportion of income imply a value for θ_κ of 1.[9]

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[8] Equation (33) can also be manipulated to replace the bus-mile service elasticity variable by the fare elasticity. For use of such formulae to analyse the benefits of changes in level of frequency see Dodgson (1986).

[9] With regard to the suggestion that the value of time might be a constant proportion of income, a survey by Hensher (1976) of time valuations, mainly from studies in the UK, the USA and Australia, noted the difficulty of deriving income-related values of time from the sample sizes available in most such studies. Of the many studies surveyed, seven had found the values of κ to be related to income: of these, two found it to be a constant proportion, two a decreasing proportion, two an increasing proportiion, and one an increasing and then decreasing proportion of income. There seems no doubt, though, that θ_κ is clearly positive and well above zero (see Transport and Road Research Laboratory, 1980, pp. 150–1).

If that is so, then θ_κ clearly exceeds the income elasticity of demand for good j, the public transport service (θ_j). Hence $\Delta_\kappa/\Delta < 1$; this implies service improvements tend to benefit the better-off more than fare reductions, because of the higher value higher income groups place on time.

5. SUMMARY AND CONCLUSIONS

This paper has derived comprehensive benefit-cost rules for transit subsidies which incorporate the overall impact of increased subsidisation on social welfare. These rules allow for the benefits to transit riders of changes in fares or service levels and for external benefits in the form of reduced highway congestion. They make it possible to allow for the distributional impact of these benefits on different income groups, and for their valuation at weights which reflect the decision-makers' evaluation of the marginal benefits of utility gains to each group. The rules also allow for the efficiency losses from the extra taxation required to fund increased subsidies, and for the distributional impact of these additional tax burdens. Finally they enable an allowance to be made for the effects of provision of financial support from higher levels of government to the local jurisdiction actually responsible for deciding the subsidy.

Using these rules, we have shown that the decision on whether or not subsidies should be used to finance reduced fares depends on:

(1) The fare elasticity of demand for transit services: the more elastic the demand, the greater will be the benefits of new trips induced by the fare reduction.

(2) The marginal costs of providing transit service: the greater these costs (and they will be particularly high if new capacity has to be provided to cope with increased demand), the lower will be the benefits of fare reductions.

(3) The cross-elasticity of demand between transit fare and use of private cars: the greater the cross-elasticity, the more likely that private traffic congestion will be reduced.

(4) Speed-flow and vehicle operating cost relationships: the greater the responsiveness of average traffic speeds to traffic flow, and of operating costs to traffic speed, the greater will be the potential benefits of reduced traffic congestion.

(5) Income elasticities of demand for transit, for urban car travel, and for the taxed good or service: the lower the income elasticity of demand for transit, the more transit is used by lower income groups, and the greater are the distributional benefits of reduced transit fares; the higher the income elasticity of demand for urban car travel, the lower are the distributional benefits of reduced highway congestion; and the lower the income elasticities of demand for taxed goods or services, the greater will be the burden of financing increased subsidies for lower income groups, and hence the higher will be the distributional costs of funding reduced transit fares.

(6) The pattern of income distribution in the city: this will affect the pattern of distributional gains and losses.

(7) The initial tax rate on the locally taxed good or service: the higher the initial tax rate, the greater the marginal excess burden of a tax increase, and hence the lower will be the net benefits of a rise in subsidy.

(8) The price elasticity of demand for the taxed good:[10] the more elastic this demand, the greater the potential marginal excess burden, and so the lower will be the net benefits of an increase in subsidy.

(9) The proportion of increased subsidies to be funded through local taxation: the greater this proportion, the greater will be the financing burden on the local community, and hence the lower will be the perceived benefits of increased subsidy from the local jurisdiction's point of view.

We have also shown that the question whether subsidies should be used to finance increased frequency levels depends on most of these factors, and also in particular on valuations of waiting time and on the elasticity of those valuations with respect to income (the higher transit riders value waiting time, the greater will be the user benefits of reduced waiting time consequent upon an increase in bus frequency), and on waiting time/headway relationships (the more sensitive the average waiting times to changes in headways, the greater will be the benefits of service improvements).

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[10] Where transit subsidies are financed through local income taxation, the appropriate elasticity is the supply elasticity of labour.

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