

A METHODOLOGICAL NOTE ON WELFARE CALCULUS

By Yuval Shilony*

Many pitfalls await the practitioner of welfare calculus. A particular problem was brought to my attention by a student who tried to find support for his argument in a paper in this Journal by Jackson (1975). The question of finding the optimal subsidy to public transport is important, and the methodology is vulnerable and warrants a note of correction. Section I presents the problem and Jackson's analysis. Section II presents my analysis, and finally a numerical example is given in Section III.

I

In the private transport market a distortion is created because car users take the price to be their private marginal cost, which is the social average cost, while they should consider also the social cost and take the price to be the higher marginal cost, taking account of the congestion they cause. A tax on private transport could remedy the situation and correct the allocation, but for some reason it is impractical. Instead a subsidy is given to public transport, and the question is what is the optimal subsidy.

Figures 1 and 2 are similar to Jackson's. In Figure 1, D_{H_1} is the demand for private transport, AC_H the average cost and MC_H the marginal cost of producing it, including congestion. Car users choose to travel H_1 car-passenger-miles, but the optimal quantity is H_1^* . The loss of welfare, or possible improvement, is W_{H_1} . In the public transit market, depicted in Figure 2, cost per unit is fixed at AC_T and the demand D_{T_1} intersects it at the quantity T_1 .

Now enters the subsidy S . In the public transit market the price falls by S and the quantity increases to T_2 . The subsidy also shifts down the demand for private transport to D_{H_2} . The new quantity is H_2 and the new price P_2 .

Jackson asserted that $W_{H_1} - W_{H_2} - W_T$ is the welfare gain from the subsidy S . No doubt it is a legitimate procedure to measure the welfare difference between two situations by measuring how far short of the maximal possible welfare (optimum) the economy is in one situation and how far short of it it is in the second, and taking the difference between the two magnitudes. But, while W_{H_1} is indeed the possible improvement before the subsidy, $W_{H_2} + W_T$ is not the possible improvement after it, so the assertion does not hold. In fact Jackson understated the benefit of a subsidy and got too small a subsidy for the optimal one.

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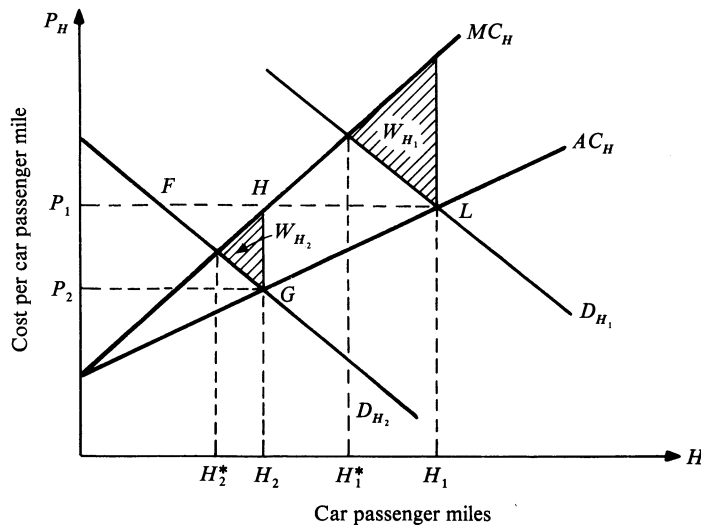


FIGURE 1
Demand and Cost Curves in the Highway Sector

II

A way to go about measuring the welfare gain is to observe separately the change for different segments of the public: consumers of various products, producers, taxpayers, etc. In our problem we observe:

- (1) In all the analysis producers' profits (or rent or surplus) are zero; they are not affected, because their price equals always the average cost.
- (2) Consumers of public transport enjoy an additional surplus represented by the area $ABCD$ in Figure 2, when the subsidy reduces the price they pay by S . That would have been all the gain if there had not been an effect on the price of H ; but
- (3) The reduction in T 's price decreases the demand for private transport to D_{H_2} and the new price is P_2 . This price reduction increases the surplus of the consumers of H by the area P_2P_1FG in Figure 1. For some discussion see, for example, Mishan (1976), chapter 7.
- (4) The reduction in the price of H decreases the demand for T to D_{T_2} and the quantity to T_3 . Taxpayers finance the subsidy $ABEJ$ and forgo alternative uses of their tax money.

In total, therefore, the net welfare gain generated by the subsidy S is $ABCD$ plus P_2P_1FG minus $ABEJ$.

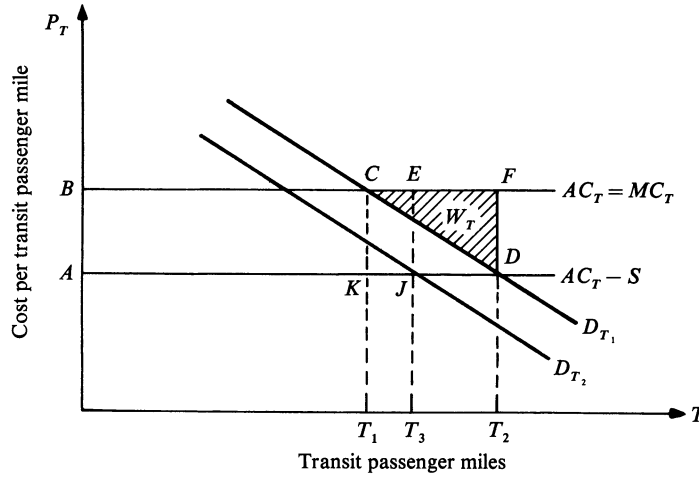


FIGURE 2
Demand and Cost Curves in the Public Transit Sector

III

A numerical example giving rise to curves similar to Jackson's will enhance the argument and, maybe, clarify it. The utility function of identical or representative consumers is

$$U(H, T, X) = 3H + 2T - H^2 - T^2 - HT + X$$

where X is a third product, "all others" serving as a numeraire. U is maximised by consumers subject to their budget constraint $P_H H + P_T T + X = I$. Plugging the constraint in U , we get

$$U(H, T) = 3H + 2T - H^2 - T^2 - HT + I - P_H H - P_T T$$

which gives rise to demand functions

$$H = \frac{4}{3} - \frac{2}{3}P_H + \frac{1}{3}P_T$$

$$T = \frac{1}{3} + \frac{1}{3}P_H - \frac{2}{3}P_T$$

which are independent of income levels. That is the strongest case for the consumer surplus as a measure of welfare. The costs of producing H and T are respectively

$$C_H(H) = \frac{1}{3}H + H^2$$

$$C_T(T) = \frac{1}{3}T$$

so $AC_H = \frac{1}{3} + H$, $MC_H = \frac{1}{3} + 2H$, $AC_T \equiv MC_T \equiv \frac{1}{3}$. The costs of production of H and T use resources that can be seen as coming at the expense of X , available initially at the quantity \bar{X} , so that $X = I - P_H H - P_T T = \bar{X} - C_H(H) - C_T(T)$. The utility of the economy as a whole \bar{U} can be written as

$$\bar{U}(H, T) = 3H + 2T - H^2 - T^2 - HT + \bar{X} - C_H(H) - C_T(T)$$

Before intervention $P_H = AC_H = \frac{1}{3} + H$, $P_T = AC_T = \frac{1}{3}$, so together with the demand functions we solve to get

$$H = \frac{11}{15}, P_H = \frac{16}{15}, T = \frac{7}{15}, P_T = \frac{1}{3} \quad \text{and} \quad \bar{U} = \bar{X} + \frac{247}{225}$$

A subsidy of, say, $\frac{1}{3}$ per unit of T means that now $P_T = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$ and still $P_H = \frac{1}{6} + H$. Now we solve and get

$$H = \frac{7}{10}, P_H = \frac{31}{30}, T = \frac{17}{30}, P_T = \frac{1}{6} \quad \text{and} \quad \bar{U} = \bar{X} + \frac{1002}{900}$$

The rise in welfare due to the subsidy is $\frac{1002}{900} - \frac{247}{225} = \frac{7}{450}$.

Using instead the procedure outlined in section II, we have

$$ABCD = ABCK + KCD = \frac{1}{6} \cdot \frac{7}{15} + \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{9} = \frac{47}{540}$$

$$P_2 P_1 FG = P_2 P_1 HG - FHG = \frac{1}{30} \cdot \frac{7}{10} - \frac{1}{2} \cdot \frac{1}{30} \cdot \frac{1}{45} = \frac{31}{1350}$$

$$ABEJ = \frac{1}{6} \cdot \frac{17}{30} = \frac{17}{180}$$

The change in welfare is $ABCD + P_2 P_1 FG - ABEJ = \frac{47}{540} + \frac{31}{1350} - \frac{17}{180} = \frac{7}{450}$ the same result.

In that explicit example the optimal subsidy is $\frac{11}{48}$ (i.e. 68% of the cost of production) which affords a utility increase of 0.0168. Jackson's analysis would recommend a subsidy of only $\frac{11}{178}$ (i.e. 18% of the cost) and a utility increase of 0.0078. The mistake implies an undervaluation of the benefit of the subsidy.

A Rejoinder

By Raymond Jackson*

In his criticism of my analysis of subsidies for public transit, Yuval Shilony claims that the derived optimal subsidy is too low because (1) the welfare loss in the subsidised transit sector is overestimated and (2) the welfare gain in the highway sector arising from the lessening of congestion is underestimated. I shall argue in Section I that Shilony's method of handling losses in the transit sector lacks a theoretical foundation and has little intuitive appeal. In the highway sector, Shilony

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suggests measuring the welfare gain as the increase in consumers' surplus arising from the reduction in the average cost per trip, while I measure the gain as the reduction in congestion costs or "dead-weight" loss. This matter is not easily settled, since the literature seems to support the use of both techniques, but Shilony appears to be correct in his advocacy of consumers' surplus. Section II discusses briefly the shortcomings of the congestion cost technique and provides a helpful warning concerning the use of consumers' surplus.

I

In the transit sector, shown in Figure 2, a subsidy S reduces the cost per trip from AC_T to $AC_T - S$ with D_{T_1} the pre-subsidy demand schedule for public transit and D_{T_2} as the final post-subsidy demand. Demand schedule D_{T_2} is shifted to the left because of the induced reduction in average highway cost per trip. In measuring the welfare loss, Shilony uses the pre-subsidy demand D_{T_1} for the increase in consumers' surplus of $ABCD$ and D_{T_2} for the calculation of the subsidy cost of $S \cdot T_3$ or $ABEJ$ to obtain a net gain of $ABCD - ABEJ$. I must object to this selective use of demand schedules and to the emphasis on the size of the subsidy rather than on the value of the incremental real resources employed. A valid procedure, as outlined by Beesley and Walters (1970), is to use the pre-subsidy demand schedule in the transit sector for both consumers' surplus and resource cost. In terms of Figure 2, a fare of $AC_T - S$ would increase transit use from T_1 to T_2 trips, holding the cost of highway travel constant. The economic value of this is calculated as T_1CFT_2 , the area swept under D_{T_1} from T_1 to T_2 , and subtracting the incremental resource cost of $AC_T(T_2 - T_1)$ or area T_1CFT_2 yields a net welfare loss of CFD . The subsidy itself represents a redistribution of income from taxpayers to transit users, and is relevant to the issue of income redistribution but not to allocative efficiency. Shilony's procedure of $ABCD - ABEJ$ reduces to $CDK - CEJK$, since $ABCK$ is common to both. There is no clear reason why this difference must be negative, though it is supposedly the measurement of a welfare loss.

II

The calculation of the welfare gain in the highway sector arising from a transit subsidy as the reduction in congestion costs is a controversial point, and Shilony has appropriately focused attention on it. The highway sector is shown in Figure 1 where, in the absence of congestion tolls, users consider average cost per trip AC_H rather than the marginal cost per trip MC_H . The result is a highway utilisation of H_1 instead of the optimal H_1^* . This non-optimal decision yields a congestion cost or "dead-weight" loss given by shaded area W_{H_1} . A transit subsidy shifts highway demand from D_{H_1} to D_{H_2} , with actual utilisation reduced from H_1 to H_2 and optimal utilisation from H_1^* to H_2^* . This subsidy has the effect of reducing the congestion cost arising from inefficient pricing from W_{H_1} to W_{H_2} , based on the difference between H_2 and H_2^* . I measured the value of the transit subsidy in this sector as the reduction in congestion costs of $W_{H_1} - W_{H_2}$. Shilony suggests that the welfare gain be measured as the increase in consumers' surplus to those who remain in this mode and find the

average cost per trip reduced from P_1 to P_2 . This consumers' surplus gain is illustrated in Figure 1 as area P_1FGP_2 .

It is not correct to assert that the gain in consumers' surplus always exceeds the reduction in congestion costs, so that optimal transit subsidies using congestion costs are consistently below those derived from consumers' surplus. Using Shilony's linear model, let the absolute slope of the demand schedules D_{H_1} and D_{H_2} be α and the slopes of AC_H schedules be β and β_2 respectively. It can be shown algebraically that the reduction in "dead-weight" loss due to a transit subsidy will be greater than the gain in consumers' surplus when $(H_1 - H_2)/H_1 > \alpha/(\alpha + \beta)$. In the illustrative case of Shilony's comment $\alpha = 2/3$ and $\beta = 1$. Applying the above condition means that the benefit of lower congestion costs will exceed the consumers' surplus gain if $(H_1 - H_2)/H_1 > 0.40$ or, in other words, if the transit subsidy reduces highway travel by at least 40%. The calculation of consumers' surplus based on the reduction in average highway costs will then be low because relatively few travellers remain, but the resulting "dead-weight" loss W_{H_2} will also be low; thus the overall reduction in congestion costs, $W_{H_1} - W_{H_2}$, will be large.

There still remains the issue of the appropriate measure of the welfare gain in the highway section. The "second-best" literature seems to lend support to a congestion cost approach. For example, in deriving a "second-best" pricing strategy Marchand (1968, p. 579) concludes that to minimise global misallocation:

The optimal toll must be such that the reduction of misallocation on the tollway when the toll is raised by an infinitesimal amount exactly offsets the corresponding increase in misallocation on the freeway.

In my original article, the technique adopted to derive optimal transit subsidies was, in the spirit of "second-best", one that effectively minimised the misallocation of resources in the transport system. However, the minimisation of misallocation measure is not necessarily consistent with the maximisation of utility. It would therefore have been better to consider consumers' surplus gains, assumed to be a valid measure of increased welfare, in the highway sector rather than to give "dead-weight" loss reductions the shift in the demand schedule.¹ The moral for transport policy may well be that it is appropriate to choose a "second-best" strategy but not a "second-best" objective.

A cautionary remark on the use of consumers' surplus, as outlined by Shilony, seems advisable. The area P_1FGP_2 is supported by Beesley and Walters (1970, p. 253) and Mishan (1976, p. 43), but is not valid when tolls are in place in the highway sector. In this case the decrease in toll revenues must be deducted from the increase in consumers' surplus, since the tolls represent a producers' surplus or quasi-rent. Mohring (1976, pp. 96-97) measures the consumers' surplus increment as P_1LGP_2 instead of P_1FGP_2 and shows that the net gain in the highway sector from a transit subsidy would be zero under an optimal toll system, a result not achieved exactly by the P_1FGP_2 procedure. In general, the net benefit in the highway sector of a transit subsidy can be measured by the increase in consumers' surplus

¹ Abouchar (1977, pp. 271-295) and Mohring (1976, pp. 100-104) discuss policies designed to minimise congestion costs but limit their analyses to cases where the demand schedule does not shift.

TABLE 1
Comparison of Optimal Fare Subsidies (S^/AC_T) using Congestion Cost Reductions and Consumers' Surplus Gains in the Highway Sector*

Cross-Elasticity of Demand	Using Congestion Cost	Using Consumers' Surplus
$\eta_{HT} = 0.2$	0.023	0.229
$\eta_{HT} = 0.4$	0.026	0.240
$\eta_{HT} = 0.6$	0.027	0.237

less the reduction in the tolls or similar tax revenues collected, whether or not the fees are optimal.

A recalculation of the optimal transit subsidies based on a gain in consumers' surplus of P_1LGP_2 in the highway sector does show that they have been underestimated.² Using the notation employed in my initial paper (Jackson, 1975), the optimal subsidy S^*/AC_T can be derived as

$$\frac{S^*}{AC_T} = \frac{\varepsilon_H \eta_{HT}}{\varepsilon_H \eta_{HT}^2 + \eta_{TT}(AC_T T_1/AC_H, H_1) + \eta_{HT}(AC_T/AC_H)} \quad (1)$$

where ε_H is the elasticity of the highway average cost schedule, η_{HT} is the cross-elasticity of demand, η_{TT} is the elasticity of demand for public transit by non-drivers, AC_H is the current average cost per highway trip, and AC_T is the cost per transit trip. Table 1 shows the optimal subsidy for public transit, using both the congestion cost and consumers' surplus measures with $\varepsilon_H = 0.20$, $\eta_{TT} = 0.20$, $AC_T/AC_H = 80/120$ and $T_1/H_1 = 0.25$, with the cross-elasticity of demand ranging from $\eta_{HT} = 0.2$ to $\eta_{HT} = 0.6$.³ The results indicate that the use of consumers' surplus points to a subsidy of approximately 25% of transit cost per trip, while the "dead-weight" loss approach only justifies a subsidy of 2.6%.

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² The optimal transit subsidy in (1) maximises the difference between the consumers' surplus gain of P_1LGP_2 in the highway sector and the allocative loss measure by CJK in the transit sector. See Mohring (1976, pp. 98-101) for the theoretical justification.

³ For the sources of those elasticity and cost estimates see Jackson (1975, p. 9).