PEAK-LOAD PRICING AND THE CHANNEL TUNNEL

A Re-examination

By Gordon Mills and William Coleman*

Glaister (1976) argued in this Journal that the Channel tunnel scheme (abandoned in 1975) would have created far too much terminal capacity, because it was intended to charge road vehicles being carried through the (rail) tunnel a price of £19, uniform throughout the year, in spite of large seasonal and weekly fluctuations in demand. Glaister goes on to calculate profit-maximising prices for each of 17 periods into which he divides the year; he shows that, with his peak-load pricing, optimal terminal capacity could be reduced by about 70%, with significant improvement in profits.

Unfortunately one of Glaister's models (that for dependent demands) is improperly formulated; this leads to prices which are unacceptably high, and exaggerates the difference between optimal capacities for peak-load pricing and uniform pricing. Furthermore, it may be argued that the calculations for uniform pricing take the uniformity principle to an unrealistic extreme, further exaggerating the difference between optimal capacities.

In this paper we present a revised model for the case of dependent demands, compare the peak-load pricing results for it with those obtained from a more accurate calculation of Glaister's other model, and suggest how the principle of "uniform" prices might be implemented in practice, thus giving a fresh comparison between uniform and peak-load pricing. We also explore the links between optimal prices and (short-run) marginal costs, and consider the implications of the various pricing policies for total (annual) traffic flow and for utilisation of capacity.

THE DATA ON DEMAND

From the official (governmental) study of the Channel tunnel proposal, Glaister extracts initial demand data for each of the 17 periods (as he explains on p. 101). For the projected charge of $\bar{p} = £19$ (the same in all periods), the demand levels $\bar{q}_i$ were forecast to be those shown in the final column of *his* Table 1. To these data Glaister fits a linear demand system:

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$$q_i = \xi_o^i + \sum_{j=1}^{17} \xi_j^i p_j \quad (i = 1, 2, \ldots 17) \tag{1}$$

where p_i is the price in period i, and q_i is the demand rate measured in vehicle units per hour.

At the initial demand points ($\bar{p}$, $\bar{q}_i$), the own-price elasticity is denoted e_{ii}, while e_{ij} denotes the cross-price elasticity which reflects the impact of p_j on q_i. These determine the coefficients:

$$\xi_j^i = \bar{q}_i \, e_{ij}/\bar{p} \quad \text{(for all } i, j\text{)} \tag{2}$$

Also, from (1),

$$\xi_o^i = \bar{q}_i - \bar{p} \sum_j \xi_j^i \quad \text{(all } i\text{)} \tag{3}$$

Glaister develops a pair of alternative models, resting on different assumptions about the values taken by these elasticities at the initial demand points.

THE MODEL FOR INDEPENDENT DEMANDS

In the first of these models (which we call Case A), each demand is supposed to be independent of the other prices; that is, all the cross-price elasticities are taken as zero. Further, Glaister quotes some evidence on *overall* price elasticities and then chooses $e = e_{ii} = -1.5$ for all i. On these assumptions, from (2),

$$\xi_j^i = 0 \quad \text{for all } i \neq j$$

$$\xi_i^i = -\frac{3}{38} \bar{q}_i \quad \text{for all } i \tag{4}$$

and, from (3),

$$\xi_o^i = \bar{q}_i - \xi_i^i \bar{p} = 2.5 \bar{q}_i \tag{5}$$

Together with Glaister's linear cost function (for the capital cost of capacity and for the various types of operating cost), this gives a quadratic profit function, to be maximised by appropriate choice of the 18 variables (namely, prices p_i and capacity q), subject to a capacity constraint for each of the 17 periods. When a quadratic programming algorithm is used, Glaister obtains the results shown in the first two columns of our Table 1. Our own calculations yield the results in the next two columns. Our optimal capacity of 768 vehicle units per hour (vuph) is almost identical with his value of 770 vuph; however, there are small but disturbing discrepancies in several of the off-peak prices. These discrepancies are now explored.

The dual values, associated with the capacity constraints, measure the rate of change of the objective function. Each dual relates to a relaxation of a *single* capacity constraint, and not to a general increase in capacity (which would be available in all periods). Thus each dual represents the rate of change of the revenue function if

TABLE 1

Prices and Duals for Independent and Dependent Demands

	Independent Demands (Case A)				*Dependent Demands*			
	Original		*Revised*		*Case B*		*Case C*	
Period	*Price*	*Dual*	*Price*	*Dual*	*Price*	*Dual*	*Price*	*Dual*
	£	£	£	£	£	£	£	£
1	26.9	188	26.9	189	56.4	170	25.5	164
2	25.6	166	25.6	166	56.0	172	24.9	153
3	25.3	968	25.3	968	54.8	1251	23.7	803
4	16.1	7	16.1	26	41.6	722	18.2	608
5	24.9	616	24.9	616	48.8	593	24.2	568
6	23.6	1586	23.6	1583	50.9	880	22.5	1355
7	16.0	0	16.0	0	42.9	2036	16.1	38
8	22.0	822	22.0	823	44.7	182	21.5	757
9	20.0	1662	20.0	1643	45.1	8	19.8	1569
10	15.7	0	16.0	0	45.0	0	16.0	0
11	19.7	0	17.7	0	51.0	0	17.7	0
12	16.0	0	16.0	0	46.0	0	16.0	0
13	16.1	0	16.0	0	44.5	0	16.0	0
14	16.0	0	16.0	0	43.9	0	16.0	0
15	16.1	0	16.0	0	29.0	0	16.0	0
16	16.8	0	16.0	0	26.9	0	16.0	0
17	15.8	0	16.0	0	15.9	0	16.0	0

(somehow) more vehicles could be carried in that period, and it relates to marginal revenue *after* netting out operating costs (but not capacity costs). As Table 1 shows, capacity is not fully used in period 7 or in periods 10 to 17. For each of these periods, a necessary condition for profit maximisation is that marginal (gross) revenue must equal marginal (operating) cost, and in any one period marginal operating cost per vehicle is a constant (here called *c*), because of the linearity of the cost function. Now gross revenue in period i is $R_i = t_i p_i q_i$ where t_i hours is the duration of period i. Thus marginal (gross) revenue is

$$\frac{dR_i}{dq_i} = t_i\left(p_i + q_i\frac{dp_i}{dq_i}\right) = t_i\,(2p_i + \xi_o^i/\xi_i^i) = t_i\left(2p_i - \frac{95}{3}\right) \tag{6}$$

by virtue of (1), (4) and (5). When marginal revenue equals the marginal operating cost per period $t_i c$,

$$p_i = \frac{1}{2}\left(c + \frac{95}{3}\right) \tag{7}$$

Now marginal cost in periods 7, 10, and 12 to 17 is simply the direct operating cost

$\gamma = 0.37$, giving $p_i = £16.018$. For period 11, output also attracts the permanent labour force cost, which (pro rata per vehicle) is $573 \div 173.2 = 3.3083$; thus here $c = £3.6783$ and (7) then yields $p_{11} = £17.672$. This analysis confirms the relevant price results from our programming calculations, and shows that, in periods when capacity is not fully used, it is optimal to charge a relatively low price, which is a price related to marginal operating cost.

A REFORMULATION OF THE MODEL FOR DEPENDENT DEMANDS

In his second model Glaister allows for dependent demands, and the system now takes the general form (1). He uses hypothetical (but plausible) data for the cross-elasticities, with values designed to give symmetry in the substitution effects (p. 106). But he supposes that all the own-price elasticities are still -1.5, apparently not realising the significance of this assumption. The effect is to portray demand as being more inelastic (overall) and to lead to "optimal" prices which are much too high.

In order to explore this point, consider variations around the initial price $p_i = £19$. We consider variations which allow the prices to remain equal to each other; that is, all $p_i = p$. Now the overall price elasticity is

$$e_i = \frac{p}{q_i} \frac{dq_i}{dp} \tag{8}$$

while, from (1),

$$\frac{dq_i}{dp} = \sum_j \xi_j^i \frac{dp_j}{dp} = \sum_j \xi_j^i \tag{9}$$

since all $p_j = p$. Thus at the initial demand point

$$e_i = \frac{\bar{p}}{\bar{q}_i} \sum_j \xi_j^i \tag{10}$$

In Case A, substituting from (4), the overall elasticity is $e_i = -1.5$ (for all i) as intended. But in the construction of Glaister's second model (here called Case B) some positive terms are added on the right-hand side of (10), while the own-price term remains as before; thus the absolute values of the overall elasticities are *greatly* reduced, most of them falling in the range -0.1 to -0.5.

As demand is now represented as being more inelastic (overall), it is hardly surprising that the model yields much higher optimal prices and lower optimal capacity. Glaister's results are reproduced in Table 1, in the column headed "Case B". (Our calculations for this case yield prices which are relatively close to Glaister's; but our result for optimal capacity is 450 vuph, whereas he quotes 520 vuph.)

Now the empirical evidence quoted by Glaister suggests an overall elasticity of -1.5, and it is this situation that he intends to analyse. Thus his formulation (Case B) does not seem to be a very useful representation. We have therefore analysed a Case C, in which the cross-price elasticities are those specified by Glaister but the own-price elasticities are adjusted so as to give in each period *an overall elasticity of* -1.5. For Case C, then, (10) is adapted to yield

$$\xi_i^i = -\sum_{j \neq i} \xi_j^i + e_i \frac{\bar{q}_i}{\bar{p}} = -\sum_{j \neq i} \xi_j^i - \frac{3\bar{q}_i}{38} \tag{11}$$

and

$$e_{ii} = \frac{\bar{p}}{\bar{q}_i} \xi_i^i = -1.5 - \frac{19}{\bar{q}_i} \sum_{j \neq i} \xi_j^i \tag{12}$$

Most of the e_{ii} then fall in the range -2.5 to -2.9. In other words, the possibility of transfer to "adjacent" periods makes the demand a good deal more sensitive to own-price than in Case A.

For Case C, the quadratic programming calculations yield the results shown in the last two columns of Table 1. The light thrown on marginal revenue is again of interest. As is shown in the Appendix, for Case C marginal revenue in any period is

$$\frac{dR}{dq_i} = t_i \,(2p_i + \xi_o^i / \sum_j \xi_j^i) \tag{13}$$

In each period in which capacity is not fully used, optimisation again requires that marginal revenue should equal marginal cost. For Case C, as for Case A, this yields $p_i = 16.018$, except in period 11, for which $p_{11} = 17.672$. This confirms some of the Case C results in Table 1.

DISCUSSION OF THE NEW RESULTS FOR PEAK LOAD PRICING

Given that the same overall elasticity is used in Cases A and C, it is not surprising that the results are broadly similar; but there are some instructive differences. With dependent demands, it is possible to use prices to transfer demand from the busiest periods to adjacent periods which are not quite so busy. In contrast, in Case A with independent demands, traffic not accommodated in the period in which it offers is lost entirely. In consequence, period 4 capacity in Case A is only just used up, whereas in Case C the price in that period is somewhat higher, because some demand has transferred from the busier periods 1, 2 and 3, for which prices are now a little lower. Overall, there is somewhat better use of capacity in Case C; optimal capacity is a little smaller, and profit a little larger, as is shown by calculations which are summarised in the first two columns of Table 2.

As Glaister mentions, it is desirable to avoid undue tariff complexity. It is now seen that seven periods in Case C (eight in Case A) have a uniform optimal price of £16. In these periods, capacity is not fully used; optimal price is related to marginal operating cost, and is significantly less than the uniform price of £19 proposed in the governmental study. For the other periods, it would not be difficult to devise groupings of periods if this were desired in the interests of tariff simplicity. For example, the tariff might comprise just five distinct prices, say £25, £22, £20, £18 and £16 (in Case C).

In Glaister's formulation of dependent demands (Case B), most of the optimal prices are in the range from £44 to £56, disturbingly higher than the uniform price

TABLE 2

Summary Results for Alternative Pricing Policies

	Peak-load pricing for profit maxn.		*Uniform pricing for profit maximisation*		*Peak lopping*
Demand model	*Case A*	*Case C*	*17 periods*	*15 periods*	*Case A*
Prices (£)	(see Table 1)	(see Table 1)	19.4	18.7	p_1: 22.5 p_2: 19.9 other: 19.4
Capacity (vuph)	768	692	1993	1653	1489
Annual traffic flow (mvu)	2.01	1.96	1.83	1.94	1.83
Annual capacity utilisation, %	30.0	32.4	10.5	13.4	14.0
Profit[a] (£ million)	29.4	29.9	22.5	25.2	25.5

[a] Before netting out capital costs other than those for the terminal facilities.

initially considered. In contrast, in our formulation (Case C), all optimal prices are relatively close to that initial price of £19, for which the demand system has been calibrated. This outcome is fortunate, for two reasons. The linearisation of the demand system is likely to be a very poor approximation when prices are as high as £50; and these very high prices of Case B would be much more likely to rouse political opposition to the idea of peak/off-peak price differentials. In contrast, the price differentials (and the absolute price levels) of Cases A and C are modest enough to make political opposition unlikely.

A RE-EXAMINATION OF THE BASE CASE OF UNIFORM PRICING

Though the price differentials introduced by peak-load pricing are relatively modest, that policy does appear to reduce greatly the required level of capacity. Before pursuing that contrast, however, we take a closer look at the uniform-pricing policy which Glaister uses as a starting point for his comparisons.

From Glaister's account (pp. 100–102), it seems that the official studies proposed a uniform price of £19, but it is not clear what capacity was proposed in those studies. The division of the year into 17 periods is Glaister's own construction, and the stated capacity of 2,053 vuph is that required to clear the market in the busiest of these 17 periods (period 1). The first question is whether, out of all possible choices of a uniform price, the uniform price of £19 is intended to maximise profits. And, if so, does that price maximise profit for the demand system *as postulated by Glaister* (including the use of 17 periods to make up the year)?

This second question may be answered by appropriate optimising calculations (which may be based on the Kuhn-Tucker conditions rather than on quadratic programming, because there are now only two policy variables). With uniform prices, the results will be the same for whichever demand system (Case A or Case C) is supposed. Summary results are shown in the third column of Table 2. The optimal price is £19.4, and this seems to suggest that Glaister's representation of the demand system is fairly close to that used in the official studies, if those studies intended £19 as a profit-maximising price.

In this solution, the percentage utilisation of capacity is so low as to cause one to doubt whether it is truly optimal to have so much capacity. However, equation (6) shows that at the price of £19.4 marginal revenue is about £7.1 per hour per unit, which is well above marginal (operating) cost in all periods. For a unit decrease in capacity (with the concomitant higher price, needed to clear the market in period 1 but used in all periods), the incremental loss of gross revenue is about £6,491, while operating cost savings are only £477. Of course, the net figure (approximately) matches the saving in annual capacity cost (£6,016), and optimality is illuminated and confirmed.

The next question arises from the very brief duration of the two busiest periods, periods 1 and 2, which comprise a two-hour span from 12 noon on Saturdays in July and August respectively; the next busiest time is period 3, all other daytime hours on Saturdays in those months. As is shown in Glaister's Table 1, for a uniform price of £19 it is optimal to install capacity of 2,053 vuph; but only 1,533 vuph is needed in period 3, and, of the extra capacity of 520 vuph, 444 vuph is used for barely 9 hours in the year and the remaining 76 vuph for barely 18 hours. Even within the context of a policy of uniform prices, it seems unlikely that it would be considered wise to provide so much extra capacity to be used for such a short time. Accordingly we now consider two alternative policies, both embodying only small departures from the philosophy of uniform prices.

The first alternative is to aggregate periods 1, 2 and 3; we assume that at a price of £19 the total flow of vehicles in the new composite period is simply the sum of the flows in the three distinct periods. This presupposes that any traffic not accommodated in periods 1 and 2 is willing to divert to period 3, that is, that it is willing to queue for later service or to arrive earlier in the day. For the resulting 15-period demand model the profit-maximising uniform price is then calculated; the results are shown in the fourth column of Table 2. The optimal capacity is greater than the flow of 1,533 vuph quoted for period 3 by Glaister, partly because the optimal uniform price turns out to be £18.7, which is below the initial price of £19.[1]

The second alternative is to use higher prices in periods 1 and 2, to smooth out demand. Among various possible *ad hoc* approaches, the one selected here is to keep the price of £19.4 for all other periods. Working with the assumption of independent demands (Case A), we then find higher prices which will clear the market in periods 1 and 2 if the installed capacity is 1,489 vuph (the amount required to match demand in period 3 when the price is £19.4). The results are shown in the last column of Table 2.

While keeping broadly to the philosophy of uniform pricing, both these policies lop

[1] For a case of welfare maximisation and independent demands, Craven (1971) offers a formal approach to the optimal division of the whole cycle into periods.

the very top off the peak; the second uses a surcharge, while the first departs from strict market-clearing and relies on traffic spilling over into period 3. As the results show, either of these modest changes leads to a large reduction in capacity and a large increase in profits compared with the results obtained from a strict interpretation of uniform pricing.

CONCLUSIONS

Glaister argued that, while uniform pricing leads to an optimal capacity of 2,053 vuph, peak-load pricing reduces this to 770 or 520 vuph (according to the demand model used). Our results show that this conclusion is somewhat exaggerated. For the demand pattern postulated by Glaister, strictly uniform pricing leads to optimal capacity of 1,993 vuph. This however makes something of a fetish out of uniform pricing; in practice it seems unlikely that so much capacity would be installed that about 25% of it would serve for no more than 18 hours in a year. A more realistic alternative is peak lopping (involving only small departures from the uniform-pricing principle); this reduces capacity to about 1,500 to 1,650 vuph. Full application of the peak-load pricing principle does yield a further large reduction in capacity, bringing optimal capacity down to about 700 to 750 vuph (depending on the demand model).

The price differentials required to achieve this reduction are relatively modest (unlike those calculated by Glaister for his model of dependent demands). Glaister does not discuss the consequences for total annual traffic flow. As seen in Table 2, the policies that bring reduced optimal capacity do *not* reduce the traffic level—indeed, full peak-load pricing leads to a slight increase, because the low marginal operating cost leads to a lower optimal price in periods when capacity is not fully used, and these periods are of great total duration. Consequently the percentage utilisation of capacity is markedly increased.

Finally, profits are increased significantly by peak-lopping, and even further increased by full peak-load pricing, though the improvements from the latter are not as great as stated by Glaister. Compared with the peak-lopping policies, full peak-load pricing reduces optimal capacity by roughly 50% and increases profit (as defined) by about 15% to 20%.

APPENDIX

Marginal Revenue in the Case of Interdependent Demands

Suppose that the i^{th} demand quantity q_i increases by a small increment Δq_i, while all other quantities remain unchanged, i.e. while $\Delta q_j = 0$ for all $j \neq i$. The set of price changes which induces this set of changes in demand quantities is denoted Δp_j (for all j).

Gross revenue is $R = \sum t_j q_j p_j$, and, ignoring the second-order terms in $\Delta q_j.\Delta p_j$, the incremental revenue is

$$\Delta R = \sum_j t_j(p_j \Delta q_j + q_j \Delta p_j)$$

For the particular increment Δq_i (with all other $\Delta q_j = 0$), this becomes

$$\Delta R = t_i p_i \Delta q_i + \sum_j t_j q_j \Delta p_j \tag{A1}$$

Using S to denote the summation term in (A1), and employing the demand system (1), from the main text, to substitute for the q_j,

$$S = \sum_j t_j q_j \Delta p_j = \sum_j t_j \xi_0^j \Delta p_j + \sum_k \sum_j p_k t_j \xi_k^j \Delta p_j$$

$$= \sum_j t_j \xi_0^j \Delta p_j + \sum_k p_k t_k \sum_j \xi_j^k \Delta p_j \tag{A2}$$

by virtue of the symmetry of the substitution effects (cf. Glaister p. 110), which ensures that $t_j \xi_k^j = t_k \xi_j^k$.

Also, taking increments in the demand system (1),

$$\Delta q_j = \sum_k \frac{\partial q_j}{\partial p_k} \Delta p_k = \sum_k \xi_k^j \Delta p_k \tag{A3}$$

and thus (A2) may be written

$$S = \sum_j t_j \xi_0^j \Delta p_j + \sum_k p_k t_k \Delta q_k$$

$$= \sum_j t_j \xi_0^j \Delta p_j + p_i t_i \Delta q_i$$

since $\Delta q_k = 0$ for all $k \neq i$.

Substitution in (A1) of this value for S yields

$$\Delta R = 2 p_i t_i \Delta q_i + \sum_j t_j \xi_o^j \Delta p_j \tag{A4}$$

and this result holds for all linear demand systems which have symmetry in the substitution effects.

Using the additional properties of the present case, the summation term in (A4) may be further refined. From (3) in the main text,

$$\xi_o^j = \bar{q}_j - \bar{p} \sum_k \xi_k^j \tag{A5}$$

and from (10),

$$\bar{q}_j = \frac{\bar{p}}{e} \sum_k \xi_k^j \tag{A6}$$

where e denotes the overall elasticity of demand, which (by assumption) is the same for all j. Thus, after substituting for $\bar{q}_j$,

$$\xi_o^j = \frac{1-e}{e}\bar{p}\sum_k \xi_k^j$$

and, on substituting this, the summation term in (A4) becomes

$$\frac{1-e}{e}\bar{p}\sum_j\sum_k t_j \xi_k^j \Delta p_j = \frac{1-e}{e}\bar{p}\sum_j\sum_k t_k \xi_j^k \Delta p_j \quad \text{from the symmetry}$$

$$= \frac{1-e}{e}\bar{p}\sum_k t_k \sum_j \xi_j^k \Delta p_j$$

$$= \frac{1-e}{e}\bar{p}\, t_i \Delta q_i$$

from (A3), and since $\Delta q_k = 0$ for $k \neq i$. Using (A5) and (A6) to eliminate $\bar{p}$ and e, this may be written alternatively as

$$t_i \Delta q_i \xi_o^i / \sum_j \xi_j^i$$

When this result has been subsituted in (A4), that equation may be divided by Δq_i, and on taking the limit as $\Delta q_i \to 0$, the result is marginal revenue

$$\frac{\partial R}{\partial q_i} = t_i(2p_i + \xi_o^i / \sum_j \xi_j^i) \qquad \text{(A7)}$$

Clearly, this result is a generalisation of equation (6) in the main text. Its simplicity is due to the assumption that the overall price elasticity is the same in all periods, when measured at a uniform price $\bar{p}$.

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