ELASTICITY MEASURES AND DISAGGREGATE CHOICE MODELS

By J. P. Dunne*

INTRODUCTION

As with any demand study, elasticity measures of the responsiveness of demand to changes in policy-relevant variables are of great importance for disaggregate choice models.[1] In these models the relevant elasticity measures are concerned with the responsiveness of the probability of choice of a particular alternative (or mode share) to a change in some attribute of that alternative. The coefficient estimates provided by the calibration of the models do not directly represent these effects. For example, in the logit model the coefficients represent the effects of a unit change in an explanatory variable on the log of the odds of choosing the particular mode. For this model, elasticity estimates are clearly of great importance in providing information for planners.[2]

Disaggregate probabilistic choice models are estimated at the level of the individual behavioural unit. This has many advantages over traditional approaches which become special cases where homogeneity within zones is assumed; but it does present problems in providing aggregate predictions, such as aggregate elasticity estimates.[3] This paper provides some comparative assessment of various available measures, concentrating on a binary logit model for simplicity and using a study of mode choice for the journey to work between Livingston New Town and Edinburgh (see Dunne, 1982) for empirical application.

The next section considers the available point and arc elasticity measures at both the individual and aggregate levels. After that we present the results of the empirical application of the various measures. The last section reviews the implications and presents some conclusions.

---

* Institute for Employment Research, University of Warwick. This paper is based on research undertaken at Edinburgh University and Birkbeck College. I am grateful to John Heath, Brian Main, John Muellbauer, Colin Roberts, Ron Smith, J. N. Wolfe and the managing editor of this journal for comments, and to the SSRC for financial support.

[1] For surveys of disaggregate choice models see, for example, Hensher and Johnson (1981), Amemiya (1981), Hensher and Dalvi (1978), Stopher and Meyburg (1976).

[2] For a discussion of these issues see, for example, Dunne (1982), Hensher and Johnson (1981), Richards and Ben Akiva (1975), Domencich and McFadden (1975). Taplin (1982) considers how ordinary demand elasticities may be inferred from choice elasticities.

[3] For a more general discussion of these issues see Dunne (1982), Koppelman (1976), McFadden and Reid (1976), Talvitie (1976).

ELASTICITY MEASURES

Within the context of disaggregate models[4] it is reasonable to consider the responsiveness of the individual choice probabilities (P_i) to a change in a specified explanatory variable (x_{1i}). This is measured by the "micro" elasticity (De Donnea, 1971), which for the logit is

$$e^P_{1i} = (1 - P_i)\beta_1 x_{1i} \tag{1}$$

Such micro elasticities are of limited use, however, as, for planning purposes, some form of aggregation of the individual measures will be required. One possible approach to deriving elasticities relating to aggregate behaviour is simply to substitute the average probability and variable values into equation (1). This "representative individual" approach is used by Richards and Ben Akiva (1975) for estimating both point and arc elasticities. It has, however, the defect that the distribution of elasticities across the sample is ignored, and the results will be biased if the sample is not composed of homogeneous individuals.

A more satisfactory approach is to consider the analogue of traditional market demand elasticities. Domencich and McFadden (1975) show that this implies an aggregate point elasticity of the form

$$E^P_1 = \frac{\Sigma P_i(1 - P_i)\,\beta_1 x_{1i}}{\Sigma P_i} \tag{2}$$

if a uniform percentage change in x_{1i} is assumed for each individual. This measure is a weighted average of the individual elasticities

$$E^P_1 = w_i e^P_{1i}$$

where the weights (w_i) are the individual's proportion of the total probability. If it is assumed that the explanatory variable change is the same for all individuals, then

$$E^P_1 = \frac{\Sigma P_i(1 - P_i)\,\beta_1 \bar{x}_1}{\Sigma P_i} \tag{3}$$

In the same way as for the micro elasticity, it is necessary to consider an arc elasticity measure for large changes in the explanatory variables. This can be defined as

[4] For a more general discussion of elasticities in transport demand models see Bly and Webster (1980).

$$E_1^{arc} = \frac{\Delta \bar{P}}{\Delta \bar{x}_1} \cdot \frac{\tilde{x}_1}{\tilde{P}} \tag{4}$$

where x_1 and P are some value between the initial and final values of x_1 and P. The arc elasticity measure can take a number of forms; these will be considered in the next section.

It is possible to devise an approximate and computationally simpler form of this arc elasticity (Tag el Din, 1980) by assuming

$$\frac{\Delta P}{\Delta x} = \frac{\partial P}{\partial x} \tag{5}$$

and substituting the values of $\partial P/\partial x$ from (2) and (3) into (4). This form has the advantage that it can be estimated without recalculation of the model probabilities. Whether or not such measures are adequate is an empirical question dependent on the acceptability of (5).

EMPIRICAL APPLICATION

The model used to provide estimates of the various elasticity measures is from a study of binary mode choice (car/bus) for the journey to work between Livingston New Town and Edinburgh (Dunne, 1982). The model was estimated for both the total sample and a sub sample containing only those individuals whose household had a car available. Results for both these samples are shown in Table 1.[5]

Table 2 gives estimates for the various aggregate point elasticities based on the final model for the total and car-available samples. Though some of the reported elasticity values have no interpretative value (for instance, those for the constant and the dummy socioeconomic variables), they are useful for purposes of illustration and comparison.

The results illustrate clearly the difference between the elasticity values and the coefficient estimates, which result from the form of the model, and show the danger of putting too much interpretative value on the coefficient estimates regarding their quantitive effect on the choice probability. The high level of overestimation of the representative individual elasticity, as expected, is apparent – its values sometimes being double that of the corresponding weighted elasticity. Further, the aggregate elasticities for both absolute and percentage change appear to be of similar magnitude, especially for the continuous (non-dummy) variables. This last feature was to be expected, given the small changes involved with point elasticities. Another interesting point is that, in this study, the average elasticity

[5] Note that the elasticities presented here consider relative time and cost rather than the actual value for the specific alternative. This means that they are not necessarily directly comparable with elasticity values presented in other studies.

TABLE 1

Final Model Forms

Constant		−1.525 (−2.147)	0.120 (0.139)
Time difference (*TIME*)		−0.030 (−1.227)	−0.011 (−0.386)
Cost difference (*COST*)		−0.072 (−1.698)	−0.085 (−1.650)
No. cars ÷ no. full time employed (*NCR*)		3.562 (6.281)	1.482 (2.173)
Social class dummy	(1 if professional)(*DSOC*) (0 otherwise)	1.533 (2.266)	1.437 (1.905)
Status dummy	(1 if head)(*DST*) (0 otherwise)	1.169 (2.963)	2.081 (4.286)
Start and finish dummy	(1 if peak)(*DSF*) (0 otherwise)	−0.869 (−2.120)	−1.099 (−2.387)
No. observations		214	175
No. iterations for convergence		7	7
Log likelihood ratio*		102.769	59.376
ρ^2†		0.374	0.316

$^*LLR(L^*) = -2 \log [L(\beta_o)/L(\beta_o, \ldots, \beta_k)] \sim \chi^2_k$

$^\dagger\rho^2 = 1 - [L^*(\beta_o, \ldots, \beta_k)/L^*(\beta_o)]$

is seen to be closer than the representative individual measure to the weighted aggregate elasticity. The increased homogeneity of the car-available sample, however, led to closer agreement between the various measures than in the total sample, especially for the representative individual elasticity.

Tables 3 and 4 give the results of various arc elasticity measures for percentage changes in the explanatory variables, together with the relevant point elasticity, the representative arc elasticity based on equation (1), and the approximate arc

TABLE 2

Elasticity Estimates

	Coefficient	Aggregate Elasticities: %	Absolute	Representative Individual	Average
Total sample					
CONSTANT	−1.525	−0.296	−0.296	−0.520	−0.520
TIME	−0.030	0.077	0.082	0.145	0.127
COST	−0.072	−0.126	0.118	−0.207	−0.237
NCR	3.562	0.321	0.443	0.779	0.384
DSOC	1.533	0.027	0.035	0.061	0.029
DST	1.169	0.122	0.148	0.261	0.164
DSF	−0.869	−0.070	−0.065	−0.115	−0.146
Car-available sample					
CONSTANT	0.120	0.018	0.018	0.027	0.027
TIME	−0.011	0.020	0.022	0.034	0.028
COST	−0.085	−0.119	−0.105	−0.161	−0.187
NCR	1.482	0.145	0.174	0.266	0.195
DSOC	1.437	0.022	0.028	0.043	0.025
DST	2.081	0.117	0.204	0.313	0.107
DSF	−1.099	−0.091	−0.068	−0.103	−0.163

elasticity of equation (7) using the linear approximation for the relevant arc elasticity form. The forms of equation (4) considered are

A linear approximation
B using the original $\bar{x}$ and $\bar{P}$ values
C using the final $\bar{x}$ and $\bar{P}$ values
D using a linear approximation for x and the value of $\bar{P}$ at that value of $x(\bar{P}^*)$

$$\frac{\Delta\bar{P}}{\Delta\bar{x}} \cdot \frac{\bar{x}_1 + (\Delta\bar{x}_1/2)}{\bar{P}^*}$$

The most striking features in these results are the closeness of the approximate and linear arc elasticities and the expected large over-estimation of the representative individual elasticity. The results for the approximate arc form are both interesting and useful; they indicate that it is possible to get a useful approximation to arc elasticity values without recomputing the probabilities.

It is clear, however, that the precision of the approximate elasticities will, by its

TABLE 3

Arc Elasticity Estimates

	Change	Average Probability	Point elasticity	Representative individual	Approximate arc	A	B	C	D
Total sample	%								
TIME	−10	0.664	0.077	0.143	0.081	0.080	0.076	0.080	0.080
	−5	0.661		0.144	0.079	0.079	0.077	0.078	0.079
	5	0.656		0.146	0.075	0.075	0.077	0.075	0.075
	10	0.654		0.147	0.073	0.074	0.077	0.074	0.074
COST	−10	0.667	−0.126	−0.202	−0.119	−0.122	−0.129	−0.121	−0.122
	−5	0.663		−0.204	−0.122	−0.125	−0.129	−0.125	−0.125
	5	0.655		−0.209	−0.129	−0.132	−0.129	−0.133	−0.132
	10	0.650		−0.212	−0.133	−0.136	−0.129	−0.137	−0.136
NCR	−10	0.636	0.321	0.831	0.332	0.331	0.343	0.337	0.331
	−5	0.648		0.804	0.326	0.326	0.332	0.329	0.326
	5	0.669		0.756	0.316	0.316	0.311	0.314	0.316
	10	0.679		0.734	0.310	0.312	0.302	0.307	0.312
Car-available sample									
TIME	−10	0.773	0.020	0.034	0.021	0.021	0.020	0.021	0.021
	−5	0.772		0.034	0.020	0.020	0.020	0.020	0.020
	5	0.771		0.034	0.019	0.019	0.020	0.019	0.019
	10	0.770		0.034	0.019	0.019	0.020	0.019	0.019
COST	−10	0.781	−0.119	−0.154	−0.113	−0.113	−0.119	−0.112	−0.113
	−5	0.776		−0.157	−0.116	−0.117	−0.120	−0.116	−0.117
	5	0.767		−0.164	−0.123	−0.124	−0.121	−0.125	−0.124
	10	0.762		−0.167	−0.126	−0.128	−0.122	−0.129	−0.128
NCR	−10	0.760	0.145	0.279	0.138	0.143	0.149	0.144	0.143
	−5	0.766		0.272	0.142	0.144	0.147	0.144	0.144
	5	0.777		0.259	0.148	0.146	0.143	0.145	0.146
	10	0.782		0.253	0.151	0.147	0.141	0.146	0.147

TABLE 4

Arc Elasticity Estimates

	Change	*Average probability*	*Representative individual*	*A*	*B*	*C*	*D*
Total sample	%						
TIME	−40	0.679	0.136	0.089	0.075	0.087	0.089
	−20	0.669	0.140	0.083	0.076	0.082	0.083
	20	0.649	0.149	0.071	0.078	0.071	0.070
	40	0.638	0.153	0.064	0.079	0.065	0.064
COST	−40	0.693	−0.186	−0.100	−0.128	−0.098	−0.100
	−20	0.676	−0.196	−0.114	−0.129	−0.113	−0.114
	20	0.642	−0.217	−0.144	−0.129	−0.145	−0.144
	40	0.625	−0.227	−0.159	−0.129	−0.163	−0.159
NCR	−40	0.548	1.033	0.369	0.422	0.407	0.365
	−20	0.610	0.890	0.343	0.367	0.357	0.342
	20	0.696	0.694	0.304	0.284	0.296	0.303
	40	0.726	0.626	0.291	0.254	0.277	0.298
Car-available sample							
TIME	−40	0.777	0.033	0.023	0.019	0.023	0.023
	−20	0.774	0.034	0.022	0.020	0.021	0.022
	20	0.768	0.034	0.018	0.020	0.018	0.018
	40	0.765	0.035	0.016	0.020	0.016	0.016
COST	−40	0.807	−0.136	−0.090	−0.115	−0.088	−0.090
	−20	0.790	−0.148	−0.105	−0.118	−0.104	−0.105
	20	0.752	−0.174	−0.137	−0.123	−0.138	−0.137
	40	0.733	−0.188	−0.154	−0.125	−0.158	−0.154
NCR	−40	0.721	0.324	0.134	0.163	0.139	0.134
	−20	0.748	0.293	0.140	0.153	0.142	0.140
	20	0.792	0.241	0.148	0.136	0.146	0.148
	40	0.811	0.219	0.151	0.129	0.147	0.151

nature, behave better as the change involved in the relevant explanatory variable becomes smaller. The level of over-estimation of the representative individual elasticity, as used by Richards and Ben Akiva (1975), illustrates the danger of using such a measure when there is heterogeneity within the sample. The results in Tables 3 and 4 also show that the degree of error involved in using a point elasticity for anything more than a 5 per cent change in the relevant explanatory variable can indeed be quite high. This error of the point elasticity is, however, lower than that reported by Richards and Ben Akiva (1975), because they were comparing it with the representative form in their study. The precision of the approximate elasticities will, however, be better in proportion to any reduction in the change in the relevant explanatory variable considered as a result of the implicit linear approximations of the logit response function. In addition, the linear arc elasticity gives the same estimates as the measure which uses the estimated probability for the value of the variable, while the measures using the original and final values of the variables and probabilities show, as expected, more discrepancy.

Results for various forms of the arc elasticities based on an equal absolute change in the relevant explanatory variables were found, not surprisingly, to be consistent with the features presented by their "percentage change" counterparts.

CONCLUSIONS

The comparative assessment of the elasticity measures presented here has provided some useful results regarding the adequacy of the commonly used aggregate forms, both point and arc. Overall, the results illustrate the dangers of using representative elasticity forms (as Richards and Ben Akiva (1975) do) when the sample is heterogeneous, and of using point elasticities to predict the effects of large changes in the relevant explanatory variable. In addition, the approximate forms of arc elasticity, with its computational advantages, were found to perform well. Clearly, the choice of measures used will involve a trade-off between bias in simple procedures (though the approximate measures presented here have performed well) and the practical costs of the more complex measures. Again, it is necessary in interpreting the measures used to bear in mind their limitations and to consider how far they are adequate for the use for which they are required.

REFERENCES

Amemiya, T. (1981): "Qualitative Response Models: A Survey". *Journal of Economic Literature*, Vol. XIX, December.

Bly, P. H., and F. V. Webster (eds) (1980): *The Demand for Public Transport.* Transport and Road Research Laboratory, Crowthorne.

De Donnea, F. X. (1971): *The Determinants of Transport Mode Choice in Dutch Cities.* Rotterdam University Press.

Domencich, T. A., and D. McFadden (1975): *Urban Travel Demand: A Behavioural Analysis.* North Holland/American Elsevier.

Dunne, J. P. (1982): *Mode Choice in New Towns: A Case Study of Livingston.* Unpublished Ph.D. thesis. University of Edinburgh.

Hensher, D. A., and M. Q. Dalvi (eds) (1978): *Determinants of Travel Choice.* Teakfield, Farnborough.

Hensher, D. A., and L. W. Johnson (1981): *Applied Discrete Choice Modelling.* Croom Helm Ltd.

Koppelman, F. S. (1976): "Guidelines for Aggregate Travel Prediction". *Transportation Research Record*, Vol. 610, pp. 19–24.

McFadden, D., and F. Reid (1976): "Aggregate Travel Demand Forecasting from Disaggregate Behavioural Models". *Transportation Research Record*, No. 534.

Richards, M. G., and M. E. Ben Akiva (1975): *A Disaggregate Travel Demand Model.* Saxon House.

Stopher, P. R., and A. H. Meyburg (1976): *Behavioural Travel Demand Models.* Lexington Books, D. C. Heath and Co.

Tag el Din, S. (1980): *A Logit Analysis of the Effect of Relocation on Job Quit Probability.* Unpublished Ph.D. thesis. University of Edinburgh.

Talvitie, A. P. (1976): "Mathematical Theory of Travel Demand" in Stopher, P. R. and A. H. Meyburg: *Behavioural Travel Demand Models.*

Taplin, J. H. E. (1982): "Inferring Ordinary Elasticities from Choice or Mode Split Elasticities". *Journal of Transport Economics and Policy*, Vol. XVI, January.