THE OPTIMAL PRICING OF FREIGHT IN COMBINATION AIRCRAFT

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In an article published in this *Journal,* Miller [1] concludes that the Civil Aeronautics Board's policy of setting belly-freight rates equal to pure freighter (average and marginal) costs is generally consistent with economic criteria for efficient pricing. The purpose of this note is to show that this conclusion is valid only in certain circumstances.

Miller derives the necessary conditions for efficient short-run pricing of air freight via a welfare maximisation model. Costs for both combination and all-freight aircraft are assumed to be functions of "transport" and "service" costs which correspond to capacity and non-capacity costs. Welfare is maximised subject to the constraint that combination aircraft are flying at optimal capacity with respect to *both* passengers and freight. On the basis of this model, Miller concludes that "the optimal level for freight rates is the average (and marginal) cost of carrying the freight in *freighter* aircraft." That conclusion is correct, given the specification of the model. However, the constraint of no excess capacity in combination aircraft is of questionable validity. With the advent of wide-bodied jets, there is considerable excess capacity for belly freight, at least along some routes. We will show that the optimal level of freight rates is not necessarily equal to the cost of carrying freight in freighters when there is excess capacity for belly freight in combination aircraft.

NOTATION AND DEFINITIONS

Except where noted, we will follow the notation and definitions used by Miller:

D_p = passenger demand for air travel, a decreasing function of X_p
D_f = demand for air freight, a decreasing function of X_f
X_p = passenger revenue ton-miles
X_{f1} = freight revenue ton-miles in combination aircraft
X_{f2} = freight revenue ton-miles in freighter aircraft
X_f = freight revenue ton-miles, total ($X_f = X_{f1} + X_{f2}$)
L_p = target (and actual) passenger load factor
L_{f1} = target (and actual) freight load factor in combination aircraft
L_{f2} = target (and actual) freight load factor in freighter aircraft
R = ratio of freight (ton-mile) capacity to passenger (ton-mile) capacity in combination aircraft

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C_p = average (and marginal) passenger traffic cost (per revenue ton-mile)
C_{f1} = average (and marginal) freight traffic cost (per revenue ton-mile) in combination aircraft
C_{f2} = average (and marginal) freight traffic cost (per revenue ton-mile) in freighter aircraft
C_a = average (and marginal) aircraft capacity cost (per available ton-mile) for aircraft of either type.

Miller defines total cost in combination aircraft as

$$K_1' = [C_p X_p + C_{f1} X_{f1}] + C_a (X_p / L_p + X_{f1} / L_{f1}) \quad (1a)$$

and in all-cargo aircraft as

$$K_2 \quad C_{f2} X_{f2} + C_a (X_{f2} / L_{f2})\ ^{1} \quad (2)$$

The first term on the right-hand side of each of the two cost equations represents total service costs, and the second term represents capacity costs, where the ratio X/L is a measure of capacity. For the discussion that follows, it is necessary to express capacity for combination aircraft as available capacity, which is a function of the type of plane and frequency of service. Define new variables T_f and T_p as the available capacity for freight and passengers in combination aircraft. Since capacity cannot be exceeded and configuration is fixed in the short run, the following conditions must hold:

$$T_f \geqslant X_{f1} / L_{f1}$$
$$T_p \geqslant X_p / L_p$$
$$T_f / T_p = R$$

The sum of T_f and T_p then represents a measure of available capacity. These three conditions imply that available capacity is a function of the configuration of the aircraft and of the quantity of the service provided at capacity. For example, if there is excess capacity for freight, available capacity is determined by the number of flights needed to serve the demand for passenger service. The cost equation for combination aircraft is now:

$$K_1 = C_p X_p + C_{f1} X_{f1} + C_a (T_p + T_f) \quad (1)$$

Following Miller's formulation, we define economic welfare as the sum of consumer and producer surpluses. Optimal prices can then be determined by maximising the following non-linear function:

$$\max W = \int_o^{x_p} D_p dX_p + \int_o^{x_f} D_f dX_f - K_1 - K_2$$

subject to:

$$X_{f1} - L_{f1} T_f \leqslant 0$$
$$X_p - L_p T_p \leqslant 0$$
$$T_p - R T_f = 0$$
$$X_p, X_{f1}, X_{f2}, T_p, T_f, Y_1, \text{ and } Y_2 \geqslant 0$$

[1] In the article ([1], p. 267), this equation is actually expressed as $K_2 = C_{f2} X_{f2} + C_a(X_f/L_{f2})$, which is probably a printing error.

The first three constraints guarantee that combination aircraft will not be carrying more than capacity. The remaining constraints are the usual non-negativity constraints on quantities and on the two Lagrangian multipliers, Y_1 and Y_2, associated with the first two constraints.

The Kuhn-Tucker conditions give the necessary and sufficient conditions for an optimal solution, given that the objective function is concave, the constraint set is convex, and at least one feasible point exists that will satisfy all the constraints as strict inequalities. There are four possible situations: neither of the first two constraints is binding, one of the first two constraints is binding, and both the first two constraints are binding. Of interest here are the situations where just the second constraint is binding and where both the first and the second constraints are binding. That is, we will be concerned with the situations that arise when where is no excess capacity for passengers in the combination aircraft.

If excess capacity does exist in passenger aircraft for freight, but not for passengers, and some air freight is shipped, the following conditions will characterise the optimal solution:

$$D_p - C_p - C_a \frac{(T_p + T_t}{X_p} \tag{3}$$

$$D_f - C_{f1} \leqslant 0 \tag{4}$$

$$(D_f - C_{f1})\, X_{f1} = 0 \tag{5}$$

$$D_f - C_{f2} - C_a / L_{f2} \leqslant 0 \tag{6}$$

$$(D_f - C_{f2} - C_a / L_{f2}) X_{f2} = 0 \tag{7}$$

Thus, in this situation, the optimal level for passenger fares is the *total* per-passenger cost of operating the plane for passengers plus the cost of providing capacity for air freight. Conditions (4) to (7) imply that air freight will be carried in both types of aircraft only if the marginal cost of freight in combination aircraft is equal to the average cost of operating on all-freight aircraft, an unlikely occurrence. It seems probable that the marginal cost of freight in combination aircraft will be less than the average cost in freighters, since capacity costs are not insignificant. In this case, all the freight would go in combination aircraft and the optimal freight rate will be the marginal cost of freight in combination aircraft; i.e., air freight services are priced as a by-product of passenger service. This is consistent with observed behaviour. For example, there is excess capacity for belly-freight in flights serving Hawaii, and there are no pure freighters serving Hawaii. However, if the average cost in freighters is less than the marginal cost for freight in combination aircraft, then all freight will go by all-cargo planes and the optimal rate will be the per-unit cost of providing air freight services in freighters.

If the average and marginal costs for both types of aircraft were not assumed to be constant but were rather (increasing) functions of quantity, conditions (4) to (7) would imply that the supply function for air freight services would be the horizontal summation of the marginal cost curve for freight in combination aircraft and the average cost function in freighter aircraft, and freight would be so allocated between the two modes that the marginal cost of freight in combination aircraft was equal to the average cost of freight in all-cargo planes.

However, if the demand for air freight is such that there is no excess capacity in combination aircraft, the conditions for efficient pricing are the same as those derived by Miller:

$$D_p - C_p - C_a \frac{(T_p + T_f - B X_f)}{X_p} \tag{8}$$

$$D_f - C_{f2} - C_a/L_{f2} = 0 \tag{9}$$

where $B = C_{f2} + C_a/L_{f2} - C_{f1}$, the difference between the average cost of carrying freight in freighters and in combination aircraft. B is necessarily non-negative if any freight is to be carried in combination aircraft. The optimal price of freight is the per-unit cost of carrying freight in all-cargo craft, and passenger fares will be either equal to or less than the per-unit cost of providing the capacity and service.

We have shown that Miller's conclusion holds only when there is no excess capacity for either passengers or belly freight in combination aircraft, which is a special rather than a general situation.

REFERENCE

[1] Miller, J. C.: "The Optimal Pricing of Freight in Combination Aircraft". *Journal of Transport Economics and Policy,* Vol. VIII No. 3, September 1973.