

DISTRIBUTIONAL EFFECTS OF MAXIMISATION OF PASSENGER MILES

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INTRODUCTION

When economists think of developing objective functions for public enterprises they usually do so from the point of view of welfare, considering public enterprises as a means of promoting allocative efficiency. The resulting strategies (for example, long-run marginal cost pricing) have not found general approval from pricing experts within the public enterprises.

In the search for a new and more stimulating objective function, maximisation of passenger miles has recently come under discussion as a policy for transport enterprises. In fact London Transport has followed this policy since 1975.

Economists usually are quite critical of this strategy. As it is not one of the usual strategies¹, economists have to compare the result of passenger-miles maximisation with the results of their usual models of public enterprise pricing. Two main questions arise:

- (a) What are the welfare losses of passenger-miles maximisation as compared with one of the usual allocative optima? This question is skilfully treated by Glaister and Collings [4].
- (b) Has passenger-miles maximisation any distributional advantages over the usual allocative optima? This question is treated in the present paper.

THE MODEL

We start from the same model as Glaister and Collings [4]. For our purpose it is, however, sufficient to assume that the revenue cost constraint is always binding. We assume also that the price of every good is lower than the threshold price that would force consumers to give up consumption. Therefore it will be sufficient to look at an enterprise dealing in two goods. (Needless to say, generalisation towards n -goods is straightforward as long as the direct maximisation of a sum of different quantities makes any sense.)

For simplicity I will deal with money fares only in the following presentation of the model. So we have to maximise an objective function

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¹Moreover an economist needs time to accept the fact that a maximisation of a sum of different quantities can be sensible under certain circumstances.

$$\text{subject to } \left. \begin{aligned} f_1(p_1) + f_2(p_2) \\ p_1 f_1 + p_2 f_2 - c_1(f_1) - c_2(f_2) = \pi \end{aligned} \right\} \quad (1)$$

where $f_i(p_i)$ are the numbers of passenger-miles provided by alternative modes or services (or peak and off-peak services of the same mode), p_i are money fares, c_i are the cost functions. $\pi \leq 0$ is either a contribution towards fixed costs or a profit that has to be achieved by the enterprise. $\pi = 0$ points to a breakeven strategy.

Differentiation with respect to the prices yields the following marginal conditions:²

$$\left. \begin{aligned} f'_1 + \lambda \{ (p_1 - c'_1) f'_1 + f_1 \} &= 0 \\ f'_2 + \lambda \{ (p_2 - c'_2) f'_2 + f_2 \} &= 0 \end{aligned} \right\} \quad (2)$$

$\lambda > 0$ being the Lagrangian multiplier. There is an immediate economic interpretation for the sign of λ : if π is changing we can interpret $\lambda = -\delta(\Sigma f_i)/\delta\pi$ as measuring the influence of the revenue-cost constraint on our objective. If a higher revenue-cost difference is required prices must increase; the quantities of the good demanded will then decrease, and so will our objective function.

By some easy transformations the marginal conditions (2) can be brought into the best form for interpreting the distributional effects of passenger-miles maximisation:

$$(p_i - c'_i)/p_i = - \frac{1}{\lambda \varepsilon_{ii}} (\lambda + \varepsilon_{ii}/p_i) \quad i = 1, 2, \quad (3)$$

and by dividing these marginal conditions

$$\frac{(p_1 - c'_1)/p_1}{(p_2 - c'_2)/p_2} = \left(\frac{\varepsilon_{22}}{\varepsilon_{11}} \right) \cdot \left(\frac{\lambda + \varepsilon_{11}/p_1}{\lambda + \varepsilon_{22}/p_2} \right) \quad (4)$$

where $\varepsilon_{ii} = (f'_i/f_i) \cdot p_i$ is the price elasticity of demand for good i in the passenger-miles optimum.³

This pricing rule can be interpreted as including allocative as well as distributional elements. To isolate the distributional from the allocative elements, we have to define an allocative basic structure from which the passenger-miles optimum is likely to deviate and which can be taken as a basis of comparison.

The usual assumptions suggest that the best basis for comparison is the consumer's surplus approach to public sector pricing, assuming the same revenue-cost constraint. This well-known approach leads to an inverse elasticity rule:

$$\frac{(p_1 - c'_1)/p_1}{(p_2 - c'_2)/p_2} = \frac{\varepsilon_{22}}{\varepsilon_{11}} \quad (5)$$

² Following Glaister and Collings [4], we have neglected cross price effects.

³ All elasticities ε_{ii} are assumed to be negative throughout this paper.

where ε_{ii} is the price elasticity of demand for good i in the allocative optimum. In what follows we will refer to it as the Baumol-Bradford optimum.

Comparing the optima with each other we immediately see that, for $(\lambda + \varepsilon_{11}/p_1)/(\lambda + \varepsilon_{22}/p_2) = 1$, maximisation of passenger miles leads to the same prices as the Baumol-Bradford optimum (under the same revenue-cost constraint) and can thus be called distributionally neutral.

In this case public pricing means that the higher the price elasticity of a good, the lower proportionately is the relative deviation of its price from its marginal cost. If prices are above marginal costs this means a kind of monopolistic accommodation to price elasticities of demand, leading to a relative cheapening of goods with higher price elasticities. If prices are below marginal costs the goods with lower price elasticities will become relatively cheaper.

In all other cases maximisation of passenger miles will lead to distributional changes of the allocative basic structure.

To compare these distributional changes with the Baumol-Bradford case, we split the price-marginal cost deviation ratio of the passenger-miles case (4) into three terms:

$$\frac{(p_1 - c'_1)/p_1}{(p_2 - c'_2)/p_2} = E \cdot D_1 \cdot D_2 \quad (6)$$

Here E is the ratio of the elasticities of the Baumol-Bradford case. D_1 represents the changes in the elasticities under maximisation of passenger miles. (D_1 is given by the ratio of the passenger-miles elasticities ratio divided by the ratio of the Baumol-Bradford elasticities.) D_2 is a further distributionally relevant factor that takes into account the influence of the revenue-cost constraint on maximisation of passenger miles (λ):

$$D_2 = \frac{(\lambda + \varepsilon_{11}/p_1)}{(\lambda + \varepsilon_{22}/p_2)} \quad (7)$$

where ε_{ii} are the passenger-miles elasticities. As these realisations of the elasticities appear in D_1 as well as in D_2 , these distributionally relevant factors are interdependent. One aspect of interdependence can easily be derived: if $D_2 = 1$, then $D_1 = 1$ must also be valid, as in such a case the passenger-miles and Baumol-Bradford elasticities coincide (the case of distributional neutrality).

We now have to define which pricing structures will be referred to as distributionally positive or negative. Distributionally positive will always mean a relative cheapening of those goods that are mainly bought by lower income classes. In the framework of our model that means that the relative deviation of price from marginal cost $(p - c')/p$ for such a good must be smaller if the price is above marginal cost, and greater if the price is below marginal cost, compared with other goods that are mainly bought by higher income classes, and of course compared with the deviation ratio of price from marginal cost of the Baumol-Bradford case.

Let us assume for the rest of the paper that good 1 is bought mainly by lower income classes and good 2 is bought mainly by higher income classes. We can then define six cases of distributionally positive and negative results, as shown in Table 1 ($D_1 \cdot D_2 = D$).

TABLE 1

	$p_i > c'_i$	$p_1 < c'_1$ $p_2 > c'_2$	$p_1 > c'_1$ $p_2 < c'_2$	$p_i < c'_i$
$D > 1$	(1) negative	—	—	(2) positive
$D < 0$	—	(3) positive	(4) negative	—
$0 < D < 1$	(5) positive	—	—	(6) negative

Of course any practical application of maximisation of passenger miles will result in a certain value of D and a certain relationship between prices and marginal costs. So in practice it always can be shown which of our six cases occurs. And so in practice we can always answer the question whether the distributional effects are positive or negative.

From the theoretical point of view, however, it is impossible to give a general answer to this question. This results from the impossibility of providing whether D_1 or D_2 will in the usual cases be smaller or greater than unity.

Let us first consider D_1 . It is a well-known and frequent assumption that goods that are mainly bought by lower income classes will have a lower price elasticity of demand ($|\varepsilon_{11}| < |\varepsilon_{22}|$); for example, that second class railway travel has a lower price elasticity of demand than first class. However, this is not necessarily true, as the example of London Transport shows: it has been empirically proved that bus is mainly used by lower income people and rail by higher income people (Gray [5]), but the price elasticity of demand for bus $|\varepsilon_{11}|$ is greater than the price elasticity of demand for rail $|\varepsilon_{22}|$ (Glaister and Collings [4]).

So in our problem we cannot even say whether E or $E \cdot D_1$ is greater or smaller than unity.

Moreover, we are quite unable to answer generally the question whether maximisation of passenger-miles will lead to good 1 having a lower elasticity than good 2 and than the elasticities in the Baumol-Bradford optimum. So it is impossible for us to say generally whether D_1 is greater or smaller than unity.

The same difficulty applies to D_2 . We do not know to start with whether $|\varepsilon_{11}| \gtrless |\varepsilon_{22}|$.

Moreover, we cannot say whether $p_1 \gtrless p_2$. (That will depend, among other things, on the specific marginal cost situation of these goods). But if we assume that not only $|\varepsilon_{11}| < |\varepsilon_{22}|$ but also $p_1 < p_2$ (for example, second and first class rail) we have to compare a ratio of two greater figures to a ratio of two smaller ones, and so we are quite unable to answer the question whether ε_{11}/p_1 will be smaller than ε_{22}/p_2 or larger.

These problems might lead us to think of eliminating the prices from our expression for D_2 by transforming equation (3) into explicit expressions for p_1 :

$$p_i = \frac{\varepsilon_{ii}}{\lambda} \frac{(\lambda c'_i - 1)}{(\varepsilon_{ii} + 1)} \quad (8)$$

and inserting into (7)

$$D_2 = \left(\frac{\lambda c'_2 - 1}{\lambda c'_1 - 1} \right) \left(\frac{\lambda c'_1 + \varepsilon_{11}}{\lambda c'_2 + \varepsilon_{22}} \right) \quad (9)$$

This formula at least seems to enable us to evaluate D_2 in the case of equal marginal costs $c'_1 = c'_2 = c'$. But unfortunately it does not.

Even if we assume that $|\varepsilon_{11}| < |\varepsilon_{22}|$ we can only arrive at the solution that D_2 can take different values:

$$\begin{aligned} D_2 &> 1 \text{ if } \lambda c' > |\varepsilon_{22}| > |\varepsilon_{11}| \\ D_2 &< 0 \text{ if } |\varepsilon_{22}| > \lambda c' > |\varepsilon_{11}| \\ 0 &< D_2 < 1 \text{ if } |\varepsilon_{22}| > |\varepsilon_{11}| > \lambda c' \end{aligned}$$

But we cannot show the distributional effects of these different cases, as *a priori* we do not know whether the prices are above or below marginal costs. This results from the fact that

$$\left(\frac{p_i - c'_i}{p_i} \right) = - \frac{1}{\varepsilon_{ii}} \left(\frac{\lambda c'_i + \varepsilon_{ii}}{\lambda c'_i - 1} \right) \quad (10)$$

depends not on $(\lambda c'_i + \varepsilon_{ii})$ only, but on the additional factor $(\lambda c'_i - 1)$, the sign of which is uncertain.

Moreover, a general interpretation of formula (8) in cases of different marginal costs is more complicated than an interpretation of formula (6). We will therefore return to the earlier one.

If we exclude the particular case $\varepsilon_{11}/p_1 = \varepsilon_{22}/p_2$ where distributional neutrality would hold, we can distinguish six different cases. For the sake of easier explanation, we reformulate

$$D_2 = \frac{\lambda - R_1}{\lambda - R_2}; \quad R_i = \left| \frac{\varepsilon_{ii}}{p_i} \right| = \left| \frac{f'_i}{f_i} \right| \quad (11)$$

Then our above-mentioned cases are shown in Table 2. (Whether prices are above or below marginal costs can always be shown by inserting $\lambda - R_i$ into equation (3)).

In each case, decreasing the revenue-cost difference π , which leads to decreasing λ , will change the pricing policy from prices both above marginal costs to one price above and one below marginal costs, until we arrive at prices below marginal costs in both cases.

Nevertheless different distributional effects can result. If $R_1 > R_2$ the relative deviation of prices from marginal costs for good 1 (lower income classes) will be smaller if prices are above marginal costs. If π decreases and so does λ , the price of

TABLE 2

<i>Value of Lagrangian</i>		<i>Value of D, p_1, p_2</i>
	(a) $R_1 > R_2$	
$\lambda > R_1 > R_2$		$0 < D_2 < 1; p_i > c'_i$
$R_1 > \lambda > R_2$		$D_2 < 0; p_1 < c'_1; p_2 > c'_2$
$R_1 > R_2 > \lambda$		$D_2 > 1; p_i < c'_i$
	(b) $R_2 > R_1$	
$\lambda > R_2 > R_1$		$D_2 > 1; p_i > c'_i$
$R_2 > \lambda > R_1$		$D_2 < 0; p_1 > c'_1; p_2 < c'_2$
$R_2 > R_1 > \lambda$		$0 < D_2 < 1; p_i < c'_i$

good 1 will as a result be the first to fall below marginal costs. All these consequences favour the good that is mainly demanded by lower income classes. If $R_2 > R_1$ just the opposite results follow. As *a priori* there is the same probability of R_1 being smaller as greater than R_2 , we cannot say generally which good will be favoured.

From the analysis of D_1 and D_2 it follows that it is impossible to give a general answer to the question whether $D_1 \cdot D_2 \geq 1$ will prevail, and so it is impossible to point to any general results for the distributional effects of maximisation of passenger miles.⁴

SOME NUMERICAL EVALUATIONS

Starting from data for London Transport, Glaister and Collings [4] have presented numerical evaluations of maximisation of passenger miles which can quite easily be applied to our question on the distributional effects of the policy. (Remember that, in this section of their paper, Glaister and Collings suppress time costs and identify generalised cost with money fare; this identification fits very well with our assumptions.)

Table 3 shows the distributional effects of passenger-miles maximisation for our six cases set out in Table 1. These results have been deduced under the following assumptions:

- Good 1 is bus-miles, good 2 rail-miles, of London Transport; good 1 is mainly demanded by lower income people, good 2 by higher income people [5].
- The basic figures for our computations are the same as in the Glaister-Collings paper, namely, p_1, p_2, λ as given in Table 1 of their paper (100. λ being denoted as "pass mark" in this table).

⁴The reader may be reminded that we have neglected cross price elasticities throughout this paper. If cross price elasticities are introduced into the analysis, D_1 and D_2 will become much more complicated (if they can be separated at all), but our result will still be valid.

TABLE 3

Profit (£m per week)	$\eta_1 = -0.65$ $\eta_2 = -0.4$				$\eta_1 = -0.8$ $\eta_2 = -0.2$			
	E	D_1	D_2	Case (Table 1)	E	D_1	D_2	Case (Table 1)
-1.0	0.0004	1247.5	-10.2	3	0.0001	57.650.-	-4.21	3
-0.8	0.014	35.57	-7.43	3	0.005	816.-	-3.75	3
-0.6	0.029	17.14	-5.3	3	0.010	320.-	-3.47	3
-0.3	0.051	9.73	-3.31	3	0.022	109.45	-3.16	3
+0.2	0.075	6.45	-1.74	3	0.031	55.16	-2.66	3
+0.3	0.101	4.72	-1.5	3	0.034	47.44	-2.64	3
0.4	0.110	4.28	-1.32	3	0.037	41.32	-2.46	3
0.6	0.129	3.50	-0.93	3	0.043	32.-	-2.33	3
0.8	0.150	2.88	-0.54	3	0.049	25.55	-2.22	3
1.0	0.176	2.30	-0.15	3	0.055	20.73	-1.97	3
1.15	0.199	1.81	+0.16	5	0.060	17.70	-1.93	3
1.25	0.225	1.34	+0.57	5	0.063	16.21	-1.80	3

- (c) The different elasticities of the passenger-miles and the Baumol-Bradford case can be computed as follows. Assuming linear demand relations $f_i(p) = \epsilon_i + \beta_i p_i$ we immediately see that

$$\epsilon_{ii} = (f'_i/f_i) p_i = \frac{p_i}{p_i + \alpha_i} \quad (12)$$

where $\alpha_i = \epsilon_i/\beta_i$. But for α_i we realise that we know one point of our demand functions ($\bar{p}_i = 4$ pence in each case) and the point elasticity in this point η_i . If we insert $\epsilon_{ii} = \eta_i$ and $p_i = \bar{p}_i$ we arrive at the result

$$\alpha_i = 4 \left(\frac{1}{\eta_i} - 1 \right) \quad (13)$$

Looking at the results we realise that an application of maximisation of passenger miles to data for London Transport bus and rail services leads to a positive distributional result. Although the accuracy of the data should not be overestimated,⁵ the direction of the result seems to be significant.

Of course these results depend on the fact that good 1 (bus), having a higher price elasticity of demand, is the one mainly demanded by lower income people. If good 1 in our case were the good that was bought by higher income people, we should arrive at just the opposite result. But of course the result does not depend only on these elasticities, but on the other data as well.

⁵ For the different cases of our computation the price elasticities of demand vary between -0.0004 and -4.2. Certainly some of these cases are not too realistic. That is especially clear in the case of $\eta_1 = -0.8$ and $\eta_2 = -0.2$, where the price elasticities of demand in the Baumol-Bradford case vary between -3.13 and -4.2 (ϵ_{11}) and between -0.0004 and -0.27 (ϵ_{22}) respectively.

CONCLUSION

Maximisation of passenger miles may perhaps be a stimulating objective function for public traffic enterprises. From the point of view of economic theory, however, it can be shown that it leads to welfare losses as compared with the usual allocative optimum at marginal cost prices.

These welfare losses might be acceptable to a public policy maker (or the management of a public enterprise) if they could be evaluated as the "price" for positive distributional effects of passenger-miles maximisation. Unfortunately, however, it is impossible to prove theoretically whether passenger-miles maximisation will have positive or negative distributional effects.

Nevertheless, in any practical application of the policy its distributional results can be computed. A numerical example using data for London Transport shows that maximisation of passenger miles would have provided positive distributional effects of bus and rail pricing for any exogenously given amount of deficit or profit for the enterprise.

REFERENCES

- [1] Baumol, W. J., and Bradford, D. F.: "Optimal Departures from Marginal Cost Pricing". *The American Economic Review*, 1970.
- [2] Bös, D.: "Cost of Living Indices and Public Pricing." *Economica*, 1978.
- [3] Feldstein, M.: "Distributional Equity and the Optimal Structure of Public Prices." *The American Economic Review*, 1972.
- [4] Glaister, S. and Collings, J. J.: "Passenger-Miles Maximisation in Theory and Practice." *Journal of Transport Economics and Policy*, September 1978 (immediately before this article).
- [5] Gray, A.: *Urban Policy*. Saxon House Studies, 1975.