MODELS FOR THE ECONOMIC EVALUATION OF ROAD MAINTENANCE

A Comment

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In a recent issue of this journal, Abelson and Flowerdew [1] (henceforth AF) make a useful contribution to a topic neglected by transport economists, viz., the economic evaluation of road maintenance strategies. In the latter parts of their paper they outline the use of integer programming models as a means for optimising expenditures on road maintenance. This note draws attention to a few important extensions to the problem, and also corrects an error in one of their models.[1]

AF outline two types of optimising models. The first is a dynamic programming model to determine the optimal maintenance expenditures for a given road (the objective is to minimise the present value of total vehicle operating costs plus maintenance costs). The second is a multi-road multi-period model for optimising the allocation of road maintenance equipment and budgets among the competing maintenance needs. Most of our remarks are directed to the initial model, the formulation of optimal maintenance strategies related to road condition and traffic volumes.

A first (and minor) point is that AF omit a potentially important cost item in road maintenance: the costs of delays to road users while maintenance activities are carried out. This can be significant [3]. The time required to perform some maintenance activities is independent of, or increased only slightly by, the degree of deterioration of the road. Therefore, incorporating delay costs to users tends to defer the optimal time to undertake maintenance activities, and thus to make periodic maintenance less frequent. Different types of maintenance activities interfere with traffic movements to different degrees; therefore the timings of maintenance activities are not affected equally. A common example of this trade-off is the choice between daytime and night-time repairs. Night-time repairs cause lower delay costs but increased labour costs.[2]

In their appendix B, AF illustrate the use of a dynamic programming model for determining the optimal maintenance strategy for a road, given the costs of various maintenance activities and traffic operating costs associated with different road conditions. But their model is restricted to deterministic cost functions, deterioration rates and traffic volumes. This is a serious shortcoming. Introducing

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[1]*Editors note:* This correction is acknowledged by the authors.

[2]There is also the public finance implication: the socially optimal maintenance decision which involves night work imposes increased financial burden on the highway authority in order to increase benefits to road users during the day.

stochastic components is a relatively straightforward management science exercise (e.g., see Derman [2]); a more important point is that recognising the uncertainty involved brings a new dimension to the optimisation problem. It is necessary to carry out road condition and traffic surveys to ascertain the condition of the road and the volume of traffic. After each inspection one decides what maintenance activity to undertake *and* how many periods to skip until the next inspection. Therefore, the costs of inspection should be an integral part of the optimising decision; that is, there should be a simultaneous solution for an optimal inspection-maintenance strategy. If we assume stochastic (Markovian) behaviour for the deterioration process (and the randomness of traffic volume is part of the probabalistic mechanism), then we can construct a Markovian inspection-maintenance model based on Klein [4]. This optimising model is richer than AF's, yet is not much more difficult to solve.[3] Such a model is set out in our appendix.

A correction is needed for AF's formulation of an integer programming model to optimise maintenance expenditures for multiple road sections. Their model is:

$$\text{Min } C = \sum_{i=l}^{n} \sum_{j=l}^{m} C_{ij}X_{ij} \qquad (1)$$

subject to:

$$\sum_{j=l}^{m} X_{ij} < 1 \text{ (for all } i) \qquad (2)$$

$$\sum_{i=l}^{n} \sum_{j=l}^{m} r_{ijkl}\, X_{ij} < R_{kl} \text{ (for all } k,l) \qquad (3)$$

$$X_{ij} = 0 \text{ or } 1 \text{ (for all } i,j) \qquad (4)$$

where C_{ij} is the total discounted cost of maintenace strategy j for road section i (with m different strategies and n road sections); $X_{ij} = 1$ if the jth strategy is selected for the ith road section and $X_{ij} = 0$ otherwise; r_{ijkl} is the amount of resource k required in year l by the jth strategy to the ith road; and R_{kl} is the total quantity of resource k available in year l. This formulation must be modified in one of two ways. If the C_{ij}'s are the absolute costs of the strategies (which appears to be AF's intention) they are all non-negative; in view of this the inequality signs in (2) are incorrect. Otherwise "$X_{ij} = 0$ for all i and j" is a feasible and trivially optimal solution. The inequality constraints (2) should be replaced by equalities:

$$\sum_{j=l}^{m} X_{ij} = 1 \text{ (for all } i) \qquad (2')$$

Alternatively, the C_{ij}'s could be interpreted as the incremental cost relative to the "do nothing" strategy; some C_{ij}'s would then be negative and AF's formulation would be valid.

AF point out that the practical usefulness of this integer programming model is doubtful, because "available computer methods will be prohibitively expensive or impossible . . . (and) the data input required is formidable" (p. 106). However, if

[3] The optimal solution is deterministic and stationary.

the allocation of scarce maintenance resources is limited to a single year (which is a standard problem facing road authorities), the optimising model is quite feasible. Let S_j represent the limited amount of specialised maintenance teams and their equipment available for carrying out the *j*th maintenance activity; r_{ij} is the amount of activity j that road section i requires; X_{ij} and C_{ij} are as defined initially. The objective function (1) and constraints (2') and (4) remain but constraint (3) becomes:

$$\sum_{i=l}^{n} r_{ij}X_{ij} \leqslant S_j \text{ (for all } j) \tag{3'}$$

This optimisation problem has a special structure known as the "generalised assignment problem". Extremely efficient algorithms are available which will handle problems involving thousands of variables (e.g., Ross and Soland [5]). Thus this formulation of the optimisation problem can be of practical use.

APPENDIX

A Markovian Decision Model for Optimal Inspection-Maintenance Strategy

This appendix applies to highway maintenance a decision model by Klein [4] (for finding an optimal inspection-maintenance strategy under Markovian deterioration).

For simplicity of exposition, the planning horizon is assumed infinite.[4] Hence, a necessary condition for a strategy to be optimal is that it minimises long-run average cost; i.e., that is the objective function.[5]

Assume that the road can be in any one of the states, 0, 1, . . . , L, where state 0 stands for "as new" condition, and state L stands for a complete failure condition. We assume that the sequence of successive states of the road (if left untouched) forms a discrete Markov chain with transition probabilities q_{ij}, $i, j = 0, 1, \ldots, L$, and would eventually reach the terminal state (for formal definitions see Klein [4]).

Let $d_{sk}(i)$ denote a decision to convert the road from its present (known) state i to state s, and to schedule the next inspection k periods from the present. The ranges of the various indices are:

$$i = 0, 1, \ldots, L, L(1), \ldots, L(\kappa - 1); \qquad k = 1, 2, \ldots, \kappa;$$
$$s \in S_i \text{ where } S_i = \{0, 1, \ldots, L - 1\} \cup \{i\}$$

where $L(m)$ means that the road has been inoperative for m time units. We assume that the longest period of time that the road can be allowed to remain inoperative is specified and denoted by $\kappa - 1$, which implies that the longest possible interval

[4] This does not imply that one actually optimises for an indefinite period into the future, but that the objective function and structure of the problem are not expected to change. Further, maintenance decisions are purely in response to the observed condition of the road and not in response to the passage of time *per se*.

[5] Minimising long-run average cost is a workable approximation to minimising the present value of total costs (see Derman [2]).

between inspections is κ. Without loss of generality we assume that repairs are made within a single period.

Since it was proved by Derman [2] that it is sufficient to consider only stationary (i.e., independent of timing) decision rules, the model is restricted to those only; however, for technical reasons the formulation allows for randomised decision rules of the form

$$D_{i\,sk} = \mathrm{P}[d_{sk}(i)]$$
$$\text{where } \sum_{s,k} D_{isk} = 1 \quad i = 0, 1, \ldots, L(\kappa - 1)$$

although Derman [2] proved the optimal solution to be deterministic.

This decision rule, in combination with the original deterioration process, gives rise to a new "controlled" Markov chain with transition probabilities

$$p_{ij},\ i, j = 0, 1, \ldots, L(\kappa - 1) \text{ such that}$$
$$p_{ij} = \sum_{s,k} D_{isk} V_{skj}$$

where

$$V_{skj} = \begin{cases} q^{(k)}_{sj} & j = 0, 1, \ldots, L \quad s = 0, 1, \ldots, L-1 \quad k = 1, 2, \ldots \kappa \\ q^{(k-n)}_{sL} & s = 0, 1, \ldots, L-1 \quad j = L(n) \quad n = 1, 2, \ldots, \kappa - 1 \quad \kappa - n > 0 \\ 1 & s = L(m) \quad m = 0, 1, \ldots, \kappa - 2 \quad j = L(n) \quad n = 1, 2, \ldots, \kappa - 1 \\ & k = n - m > 0 \quad j = s + k \quad k = 1, 2, \ldots, L(\kappa - 1) - s \\ 0 & \text{otherwise} \end{cases}$$

Let t_{skj} be the cost of *inspection* when the actual road state is j, given that k periods ago the state was s (where if j is a failure state it includes an element of "penalty"). Let C_{isk} be the *maintenance* cost associated with the decision $d_{sk}(i)$. Note that, since both inspection and maintenance costs are assumed to depend on the length of the uninspected interval k, one of these costs must include the costs to users (calculated as the present discounted value of vehicle operating costs for the length of time the road is expected to be in each state during the intermediate period).

As stated earlier, our criterion is to minimise the long-run average cost per unit time, denoted by θ. It is clear that θ is equal to the average cost per inspection divided by the average time between inspections. The ergodic theorem for Markov chains allows us to compute the average cost per inspection, denoted by EC, according to the formula

$$EC = \sum_{i} \sum_{s,k} \sum_{j} \P_i D_{isk} V_{skj} t_{skj} + \sum_{i} \sum_{s,k} \P_i D_{isk} C_{isk}$$

where the $\P_i$'s represent the equilibrium probabilities of the different states for the control process and satisfy the conditions

$$\P_j = \sum_{i} \P_i P_{ij} \quad j = 0, 1, \ldots, L(\kappa - 1) \tag{5}$$

$$\sum_{j} \P_j = 1$$

If we represent the average time between inspections by EI, it may be computed according to the relationship:

$$EI = \sum_i \sum_{s,k} k \P_i D_{isk}$$

Thus, our criterion becomes

$$Min\ \theta = \frac{\sum_i \sum_{s,k} \sum_j \P_i D_{isk} V_{skj} t_{skj} + \sum_i \sum_{s,k} \P_i D_{isk} C_{isk}}{\sum_i \sum_{s,k} k \P_i D_{isk}}$$

Now, let $X_{isk} = \P_i D_{isk}$.

Then our criterion may be written in the form

$$Min\ \theta = \frac{\sum_i \sum_{s,k} X_{isk} h_{isk}}{\sum_i \sum_{s,k} k X_{isk}} \qquad (6)$$

where $h_{isk} = \sum_j t_{skj} V_{skj} + C_{isk}$.

Further, since $\P_j = \sum_i \P_i P_{ij} = \sum_i \P_i \sum_{s,k} D_{isk} V_{skj} = \sum_i \sum_{s,k} X_{isk} V_{skj}$

and $\P_j = \sum_{s,k} X_{jsk}$, the $L + \kappa + 1$ conditions (5) become

$$\sum_{s,k} X_{jsk} = \sum_i \sum_{s,k} X_{isk} V_{skj} \quad j = 0, 1, \ldots, L(\kappa - 1) \qquad (7)$$

$$\text{and } \sum_j \sum_{s,k} X_{jsk} = 1 \qquad (8)$$

Thus our problem is to find $\{X_{jsk}\}, j = 0, 1, \ldots, L(\kappa - 1)$ $s \in S_j$, $k = 1, 2, \ldots \kappa$ to minimise the non-linear function (6) subject to the $L + \kappa + 1$ linear constraints (7) and (8).

There are methods of converting such a fractional program to a linear program (see [2] and [4]) which can be solved by standard techniques. After solving for the optimal $\{X_{jsk}\}$ we compute the optimal D_{isk}, and, as stated before, the solution will be deterministic, i.e. D_{isk} will equal 1 for one pair (s,k) and 0 for all other pairs.

REFERENCES

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